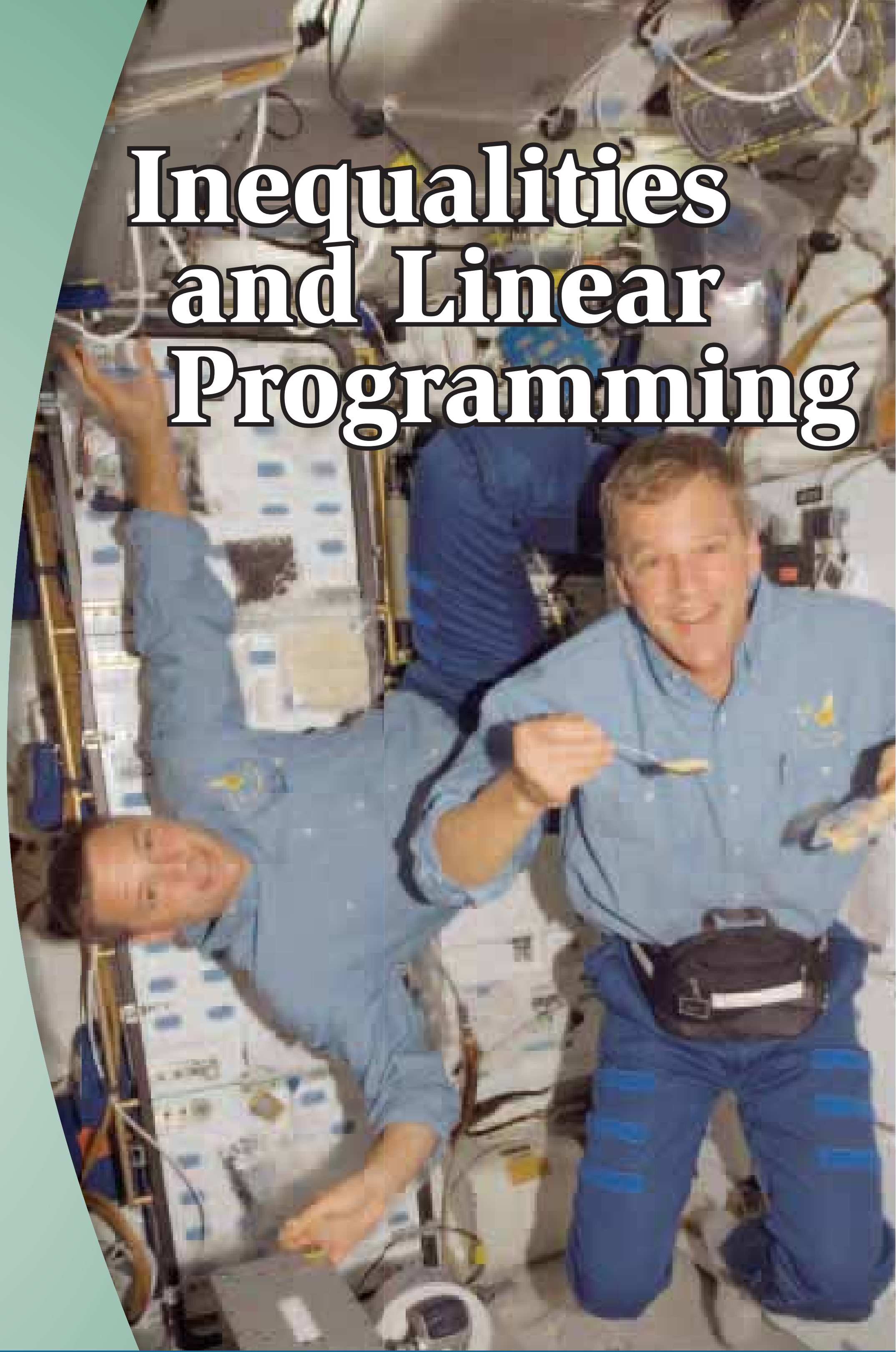
UNIT 2

Inequalities and Linear Programming



For some people, athletes and astronauts in particular, selection of a good diet is a carefully planned scientific process. Each person wants maximum performance for minimum cost. The search for an optimum solution is usually constrained by available resources and outcome requirements.

The mathematics needed to solve these and other similar optimization problems involves work with *inequalities* and a technique called *linear programming*. The essential understandings and skills required for this work are developed in two lessons of this unit.

Lessons

1 *Inequalities in One Variable*

Use numeric and graphic estimation methods and algebraic reasoning to solve problems that involve linear and quadratic inequalities in one variable.

2 *Inequalities with Two Variables*

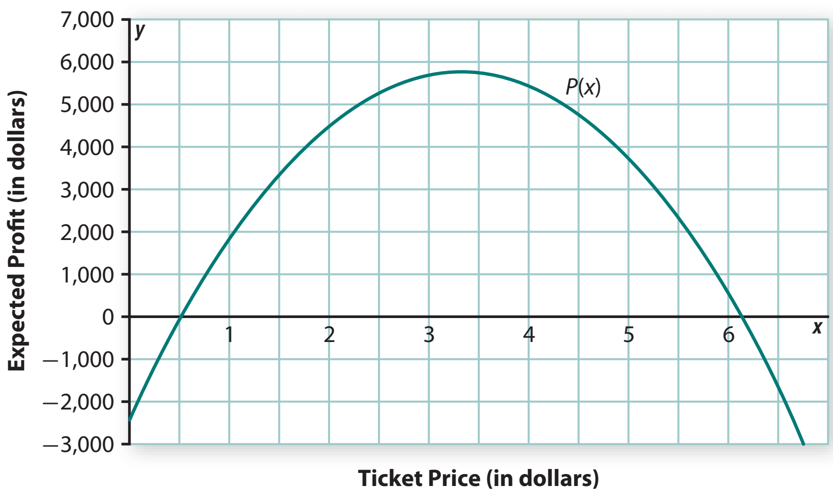
Use graphic and algebraic methods to determine solution sets for systems of linear inequalities in two variables. Recognize problems in which the goal is to find optimum values of a linear objective function, subject to linear constraints on the independent variables. Represent both objective and constraints in graphic and algebraic form, and use linear programming techniques to solve the optimization problems.

LESSON 1

Inequalities in One Variable

In previous courses, you learned how to solve a variety of problems by representing and reasoning about them with algebraic equations and inequalities. For example, suppose that plans for a fundraising raffle show that profit P will depend on ticket price x according to the function $P(x) = -2{,}500 + 5{,}000x - 750x^2$. A graph of profit as a function of ticket price is shown here.

Raffle Fundraiser Profit



Think About This Situation

Questions important to the fundraising group can be answered by solving inequalities involving the profit function.

a What would you learn from solutions of the following inequalities?

i. $-2{,}500 + 5{,}000x - 750x^2 > 0$

ii. $P(x) < 0$

iii. $-2{,}500 + 5{,}000x - 750x^2 \geq 4{,}000$

iv. $P(x) \leq 2{,}500$

b How could you use the graph to estimate solutions of the inequalities in Part a?

c In what ways could you record solutions of the inequalities in words, symbols, or diagrams?

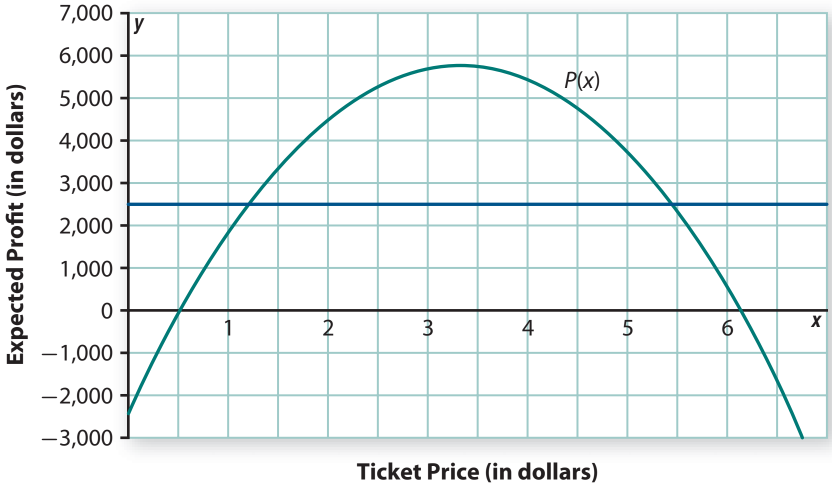
In this lesson, you will learn how to use graphical reasoning and algebraic methods to solve inequalities in one and two variables. You will also learn how to represent the solutions symbolically and graphically and how to interpret them in the contexts of the questions that they help to answer.

Investigation 1 Getting the Picture

You learned in earlier work with inequalities that solutions can be found by first solving related equations. For example, in the raffle fundraiser situation, the solutions of the equation

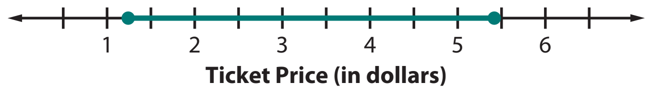
$$-2{,}500 + 5{,}000x - 750x^2 = 2{,}500$$

are approximately \$1.23 and \$5.44. The reasonableness of these solutions can be seen by scanning the graph of the profit function and the constant function $y = 2{,}500$.



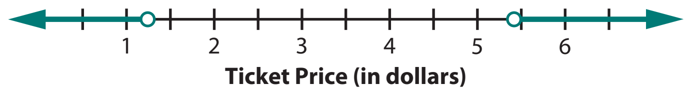
The solutions of the inequality $-2{,}500 + 5{,}000x - 750x^2 \geq 2{,}500$ are all values of x between \$1.23 and \$5.44. Those solutions to the inequality can be represented using symbols or a *number line graph*.

$$1.23 \leq x \leq 5.44$$



Similarly, the solutions of the inequality $-2{,}500 + 5{,}000x - 750x^2 < 2{,}500$ are all values of x that are either less than \$1.23 or greater than \$5.44. Those solutions can also be represented using symbols or a number line graph.

$$x < 1.23 \text{ or } x > 5.44$$

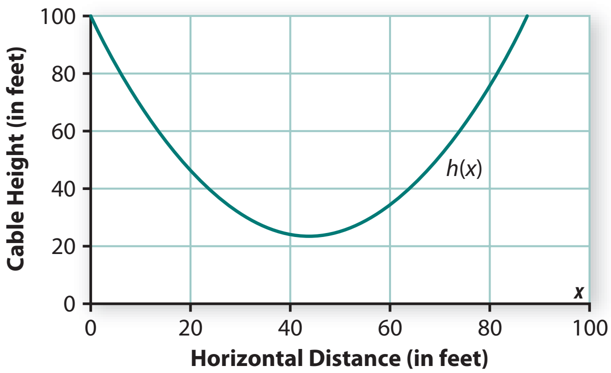


As you work on problems of this investigation, look for answers to these questions:

How can you solve inequalities in one variable?

How can you record the solutions in symbolic and graphic form?

1 The next graph shows the height of the main support cable on a suspension bridge. The function defining the curve is $h(x) = 0.04x^2 - 3.5x + 100$, where x is horizontal distance (in feet) from the left end of the bridge and $h(x)$ is the height (in feet) of the cable above the bridge surface.



For the questions in Parts a–d:

- Write an algebraic calculation, equation, or inequality whose solution will provide an answer to the question.
- Then use the graph above to estimate the solution and calculator- or computer-generated tables and graphs of $h(x)$ to sharpen the accuracy to the nearest tenth.
- Express your answer with a symbolic expression and (where appropriate) a number line graph.

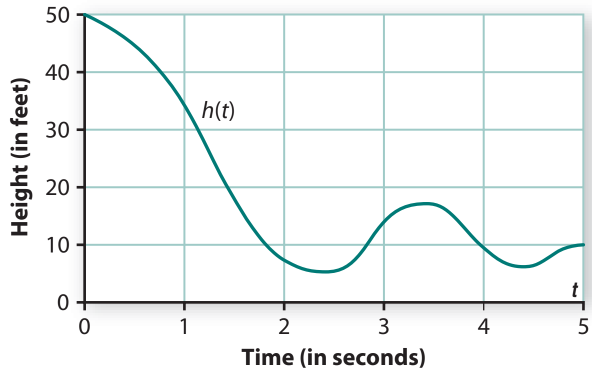
a. Where is the bridge cable less than 40 feet above the bridge surface?

b. Where is the bridge cable at least 60 feet above the bridge surface?

c. How far is the cable above the bridge surface at a point 45 feet from the left end?

d. Where is the cable 80 feet above the bridge surface?

2 The graph below shows the height of a bungee jumper's head above the ground at various times during her ride on the elastic bungee cord. Suppose that $h(t)$ gives height in feet as a function of time in seconds.



For each Part a–d:

- Write a question about the bungee jump that can be answered by the indicated mathematical operation.
- Use the graph to estimate the answer.
- Express your answer (where appropriate) with a number line graph.

a. Evaluate $h(2)$.

b. Solve $h(t) = 10$.

c. Solve $h(t) \geq 10$.

d. Solve $h(t) < 10$.

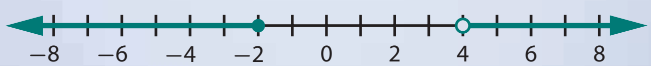
Summarize the Mathematics

In this investigation, you developed strategies for solving problems by estimating solutions for equations and inequalities in one variable. You also used symbols and number line graphs to record the solutions.

a Describe strategies for solving inequalities in the form $f(x) \leq c$ and $f(x) \geq c$ when given a graph of the function $f(x)$.

b If the solution of an inequality is described by $2 \leq x$ and $x < 5$, what will a number line graph of that solution look like?

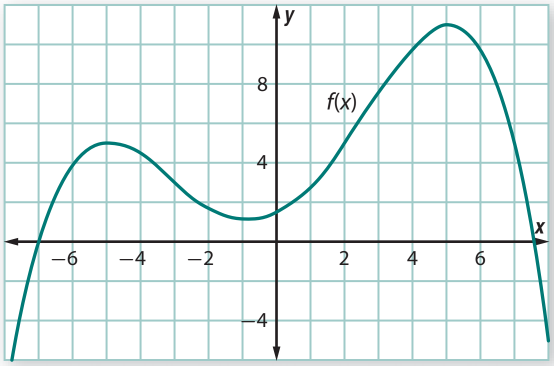
c What inequality statement(s) describe the numbers represented in this number line graph?



Be prepared to share your ideas and reasoning with the class.

Check Your Understanding

Use information from this graph of the function $f(x)$ to answer the questions that follow.



a. Estimate the values of x for which $f(x) = 5$.

b. Describe the values of x for which $f(x) \geq 5$ using the following.

i. words **ii.** symbols **iii.** a number line graph

c. Explain how your answers to Part b would change if you were asked to consider the values of x for which $f(x) > 5$?

Investigation 2 Quadratic Inequalities

Inequalities that involve functions with familiar rules can often be solved by algebraic reasoning. As you work on the problems of this investigation, look for answers to these questions:

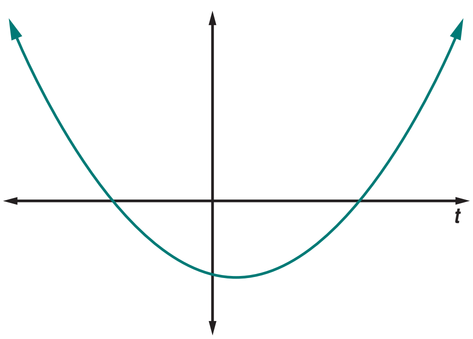
What are the solution possibilities for quadratic inequalities?

How can solution strategies for quadratic equations be applied to solution of inequalities?

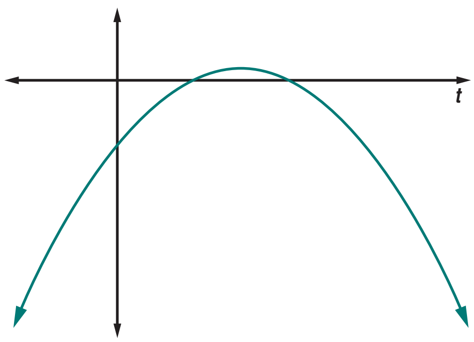
1 Consider the inequality $t^2 - t - 6 \leq 0$.

a. Which of these diagrams is most like what you would expect for a graph of the function $g(t) = t^2 - t - 6$? How can you decide without using a graphing tool?

Graph I



Graph II



b. The expression $t^2 - t - 6$ can be written in equivalent factored form as $(t - 3)(t + 2)$. How can this fact be used to solve the equation $t^2 - t - 6 = 0$? What do those solutions tell about the graph of $g(t)$?

c. Use your answers from Parts a and b to solve the inequality $t^2 - t - 6 \leq 0$. Describe the solution using symbols and a number line graph.

d. Use similar reasoning to solve the inequality $t^2 - t - 6 > 0$ and record the solutions using symbols and a number line graph.

2 Your answers to the questions in Problem 1 show how two key ideas reveal solutions to any quadratic inequality in the form $ax^2 + bx + c \leq 0$ or $ax^2 + bx + c > 0$.

a. How does sketching the graph of $f(x) = ax^2 + bx + c$ help in solving a quadratic inequality in the form above?

b. How does solving the equation $ax^2 + bx + c = 0$ help in solving a quadratic inequality like those shown?

To use these strategies effectively, you need to recall the cues that tell shape and location of a quadratic function graph and the techniques for solving quadratic equations. Problems 3–5 provide review of the ideas required by those tasks.

3 For each of these quadratic functions, use the coefficients of the terms to help determine the shape and location of the graph.

- whether the graph has a maximum or minimum point
- the location of the line of symmetry
- where the graph will cross the y-axis

a. $f(x) = x^2 - 4x - 5$

b. $g(x) = -x^2 + 2x + 8$

c. $h(x) = x^2 + 6x + 9$

d. $j(x) = 2x^2 + 2x + 1$

4 What are the possible numbers of x-intercepts for the graph of a quadratic function? Sketch graphs to illustrate each possibility.

5 Solutions for quadratic equations can be estimated by scanning tables or graphs of the related quadratic functions. Solutions can also be found exactly by factoring the quadratic expression or by use of the quadratic formula. Practice using those exact solution strategies to solve these quadratic equations. Be prepared to explain your choice of strategy.

a. $x^2 + 5x + 4 = 0$

b. $x^2 + 2x - 8 = 0$

c. $-x^2 + 6x - 9 = 0$

d. $2x^2 + 2x + 1 = 0$

e. $x^2 - 6x + 11 = 2$

f. $x^2 + 1 = -3x$

6 Combine algebraic and graphic reasoning to solve the following inequalities. For each inequality:

- sketch a graph showing the pattern of change you expect for the function involved.
- use algebraic reasoning to locate key intersection points.
- combine what you learn from your sketch and algebraic reasoning to solve the inequality.
- record the solution using symbols and number line graphs.

a. $x^2 + 2x > 0$

b. $x^2 + 2x < 0$

c. $n^2 + 2n - 24 \leq 0$

d. $n^2 + 2n - 24 > 0$

e. $-s^2 + 4s - 6 < 0$

f. $-s^2 + 4s - 6 > 0$

g. $8r - r^2 \geq 15$

h. $3x^2 - 5x > 8$

i. $z^2 - 6z + 7 < 2$

j. $-p^2 + 10p - 7 \leq 14$

Summarize the Mathematics

In this investigation, you developed strategies for solving quadratic inequalities.

a Describe strategies for solving inequalities in the form $ax^2 + bx + c \leq d$ and $ax^2 + bx + c \geq d$ by algebraic and graphic reasoning.

b What are possible solutions for quadratic inequalities, and how can each form be expressed in algebraic and number line graph form?

Be prepared to share your ideas and reasoning with the class.

Check Your Understanding

For each inequality: (1) sketch a graph of the function involved in the inequality, (2) use algebraic reasoning to locate x-intercepts of the graph, (3) combine what you learn from your sketch and algebraic reasoning to solve the inequality, and (4) record the solution using symbols and a number line graph.

a. $k^2 - 3k - 4 > 0$

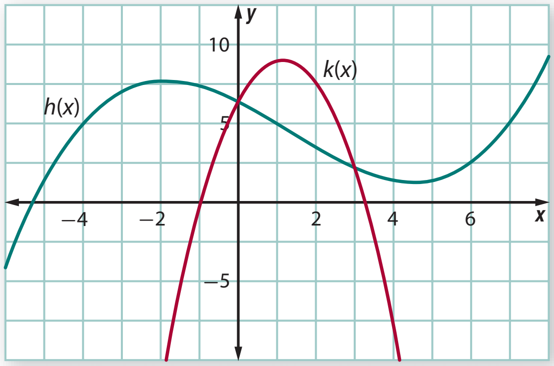
b. $-b^2 + 8b - 10 \geq 0$

Investigation 3 Complex Inequalities

Many problems require comparison of two functions to find when values of one are greater or less than values of the other. As you work on the problems of this investigation, look for answers to this question:

How can the reasoning developed to deal with inequalities involving a single function be adapted to find solutions for more complex cases?

1 The diagram below shows the graphs of functions $h(x)$ and $k(x)$. Assume that all points of intersection are shown and that the functions have no breaks in their graphs.



a. What are the approximate values of x for which $h(x) = k(x)$?

b. What are the values of x for which $h(x) \leq k(x)$? Express your answer using symbols and a number line graph.

c. What are the values of x for which $h(x) > k(x)$? Express your answer using symbols and a number line graph.

2 Harriet Tubman Elementary School needs to hire staff for a new after-school program. The program has a budget of \$1,000 per week to pay staff salaries.

The function $h(p) = \frac{1{,}000}{p}$ shows how the number of staff that can be hired depends on the weekly pay per staff member p. Research suggested that the number of job applicants depends on the weekly pay offered according to the function $a(p) = -5 + 0.1p$.

a. Without using a graphing calculator, sketch a graph showing how $h(p)$ and $a(p)$ depend on p.

b. Solve the equation $\frac{1{,}000}{p} = -5 + 0.1p$ algebraically. Then explain what the solution tells about the staffing situation for the after-school program at Harriet Tubman Elementary.

c. The program director wants to ensure that she will be able to choose her staff from a large enough pool of applicants.

i. Write an inequality that can be solved to determine the weekly pay per staff member for which the number of applicants will be more than the number of staff that can be hired.

ii. Use your responses to Parts a and b to solve the inequality.

3 For each inequality given below:

- sketch a graph showing how you expect the two component functions to be related.
- use algebraic reasoning, a computer algebra system (CAS), or estimation using function tables or graphs to locate the points of intersection of the graphs.
- apply what you learn about the relationship of the functions to solve the inequality.
- record the solution using symbols and number line graphs.

a. $q^2 + 3q - 6 \leq q + 2$

b. $c^2 - 4c - 5 \geq 2c + 2$

c. $v^2 - v + 3 > 2v - 1$

d. $2m + 3 > 4 - m^2$

e. $7 - x < \frac{10}{x}$

f. $\sqrt{d} > 2d - 1$

Interval Notation The solution of an inequality in one variable is generally composed of one or more intervals on a number line. Mathematicians use *interval notation* as a kind of shorthand to describe those solutions. For example, consider your response to Problem 1 Part b.

$$h(x) \leq k(x) \text{ when } 0 \leq x \leq 3.$$

Using interval notation,

$$0 \leq x \leq 3 \text{ can be written as } [0, 3].$$

The square brackets [,] indicate that the endpoints 0 and 3 are included in the solution set as well as all points between those end points. On the other hand, in Part c of Problem 1, you found that:

$$h(x) > k(x) \text{ when } x < 0 \text{ or } x > 3.$$

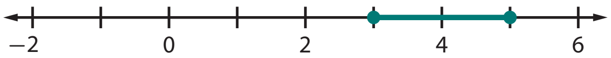
Using interval notation,

$$x < 0 \text{ or } x > 3 \text{ can be written as } (-\infty, 0) \cup (3, +\infty).$$

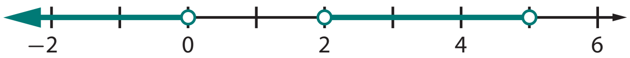
Here, the round brackets (,) indicate that the end points are not included in the solution set. The symbol ∞ represents infinity. Round brackets are always used with $-\infty$ and $+\infty$. The symbol $\cup$ indicates the *union* of the two intervals, that is, the numbers that are in one interval *or* the other. If an inequality has no real number solutions, the solution set is the *empty set*, denoted by the symbol $\varnothing$.

4 Here are descriptions of solutions for several inequalities. Describe each solution using interval notation.

a. $x < -2$ or $x > 0$

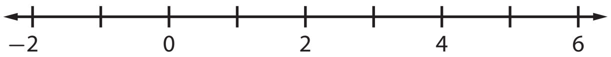
b. 

c. $0 \leq x < 1$

d. 

e. $x \leq -1$ or $x \geq 7$

f. The inequality is true for all values of x.

g. 

Summarize the Mathematics

In this investigation, you developed a strategy for solving complex inequalities in one variable. You used symbols, number line graphs, and interval notation to record the solution sets.

a Describe a general strategy for solving an inequality $f(x) < g(x)$.

b Suppose that $f(x) \geq g(x)$ for values of x from a up to and including b, where b is greater than a. Represent the solution using symbols, a number line graph, and interval notation.

c Suppose that $h(t) < j(t)$ for values of t less than c or greater than d where d is greater than c. Represent the solution using symbols, a number line graph, and interval notation.

Be prepared to share your ideas and reasoning with the class.

Check Your Understanding

For each inequality: (1) sketch a graph of the functions involved in the inequality, (2) use algebraic reasoning to locate intersection points of the graphs, (3) combine what you learn from your sketch and algebraic reasoning to solve the inequality, and (4) record the solution using symbols, a number line graph, and interval notation.

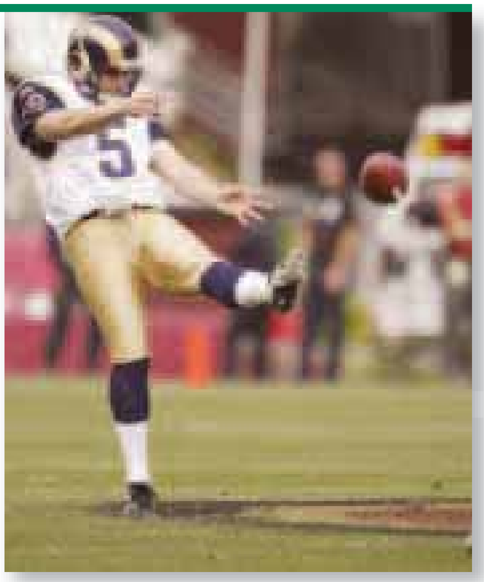
a. $2 - w \leq w^2 - 2w$

b. $\frac{6}{x} < x + 5$

On Your Own

Applications

1 The graph below shows the path of a football kick, with height above the ground a function of horizontal distance traveled, both measured in yards. The function defining the path is $h(x) = -0.02x^2 + 1.3x + 1$.



For each of the questions in Parts a–d:

- Write an algebraic calculation, equation, or inequality whose solution will provide an answer to the question.
- Use the graph above to estimate the solution and calculator-generated tables and graphs of $h(x)$ to sharpen the accuracy to the nearest tenth.
- Express your answer with a symbolic expression and a number line graph.

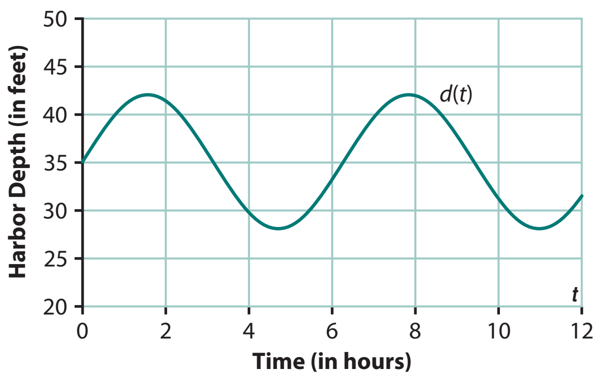
a. When is the kicked ball 20 yards above the field?

b. When is the ball less than 10 yards above the playing field?

c. How far is the ball above the field when it has traveled horizontally 40 yards?

d. When is the ball at least 15 yards above the field?

2 The next graph shows the depth of water alongside a ship pier in a tidal ocean harbor between 12 A.M. and 12 P.M. on one day. Suppose that $d(t)$ gives depth in feet as a function of time in hours.



For each Part a–d:

- Write a question about the water depth that can be answered by the indicated mathematical operation.
- Use the graph to estimate the answer.
- Express your answer (where appropriate) with a number line graph.

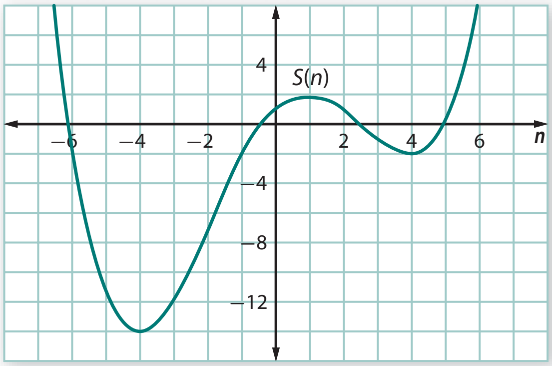
a. Evaluate $h(2)$.

b. Solve $h(t) = 40$.

c. Solve $h(t) \geq 40$.

d. Solve $h(t) < 30$.

3 Use this graph of a function $S(n)$ to estimate the values of n that satisfy each inequality below. Describe those values using words, symbols, and number line graphs.



a. $S(n) \geq -2$

b. $S(n) < -2$

c. $S(n) > -14$

d. $S(n) \leq -14$

4 Describe the solutions of these inequalities using symbols and number line graphs.

a. $7t - t^2 < 0$

b. $a^2 + 4a \geq 0$

c. $h^2 + 2h - 3 \leq 0$

d. $x^2 + 0.5x - 3 > 0$

e. $-d^2 - 12d - 20 > 0$

f. $3 + 2r - r^2 \geq 0$

5 Describe the solutions of these inequalities using symbols and number line graphs.

a. $7t - t^2 < 10$

b. $a^2 + 4a \geq 12$

c. $5h^2 + 14h < 3$

d. $-x^2 + 6x - 8 < 1$

e. $-d^2 - 11d - 20 > 4$

f. $3 + 2r - r^2 \geq 8$

6 Describe the solutions of these inequalities using symbols and a number line graph.

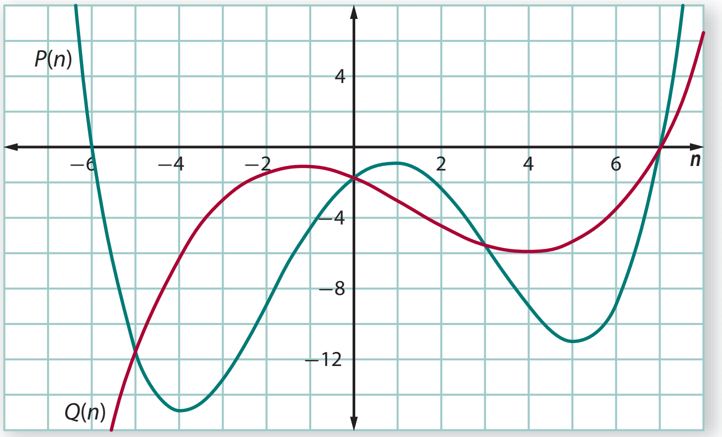
a. $k + 10 \geq \frac{24}{k}$

b. $w - 1 < \frac{20}{w}$

c. $x + 6 < \frac{7}{x}$

d. $u + 1 \leq \frac{12}{u}$

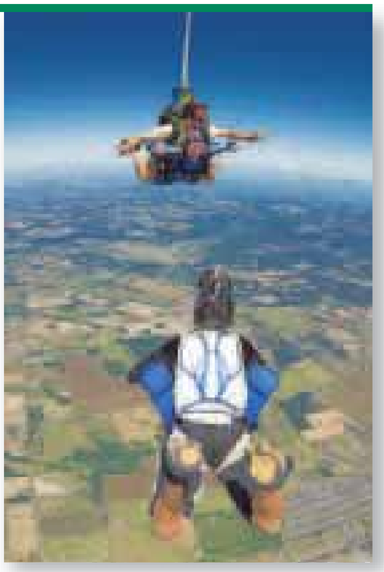
7 Shown below are the graphs of two functions $P(n)$ and $Q(n)$. Assume that the graph shows all points of intersection of $P(n)$ and $Q(n)$.



CPMP-Tools

a. Describe the values of n for which $P(n) < Q(n)$ using symbols and a number line graph.

b. Describe the values of n for which $P(n) > Q(n)$ using symbols and a number line graph.



8 The Riverdale Adventure Club has planned a skydiving lesson and first jump for new members. They plan to make and sell videos of each jump.

Income from sale of videos of the jump and expenses for producing the videos will depend on price p charged according to these rules.

$$I(p) = p(118 - p)$$

$$E(p) = 6{,}400 - 50p$$

a. Sketch graphs showing how you expect the functions $I(p)$ and $E(p)$ to be related.

b. Find coordinates of the intersection points algebraically.

c. Describe the solution of the inequality $I(p) \geq E(p)$ using symbols and a number line graph.

d. Explain what your work suggests about how much the club should charge for videos of the jump.

9 For each of the following inequalities:

- sketch graphs showing how you expect the two functions in the inequality to be related.
- use algebraic reasoning, a CAS, or estimation using tables and graphs of the functions to locate the points of intersection of the graphs.
- use what you learn about the relationship of the functions to solve the inequality.
- record the solution using symbols and a number line graph.

a. $2n^2 + 3n < 8n + 3$

b. $9 + 6x - x^2 \leq x - 5$

c. $d^2 - 5d + 10 \geq 1 - 2d$

d. $j^2 + 7j + 20 < -j^2 - 5j$

e. $5 - u^2 \geq u^2 - 3$

f. $\sqrt{b} > 6 - b$

10 Graph these intervals on number lines. Write inequalities that express the same information.

a. [−2, 5) **b.** (−∞, 0) ∪ [4, ∞)

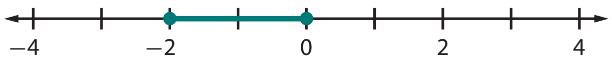
c. [3, 7] **d.** (−4, −1) ∪ (1, ∞)

11 Below are descriptions of the solutions for six inequalities. Describe each solution using interval notation.

a. $k \leq -3$ or $k > -1$

b. All numbers between negative 1 and positive 3.5

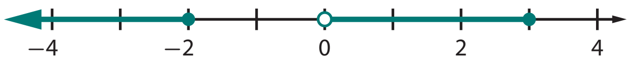
c.



d. $2 < g < 6$

e. All numbers less than 4 or greater than 7

f.



Connections

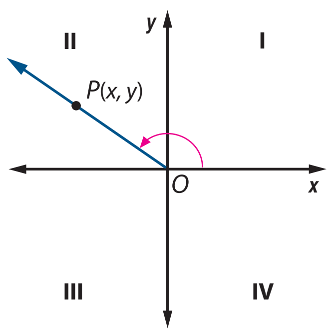
12 A firework contains a time-delay fuse that burns as the firework soars upward. The length of this fuse must be made so that the firework does not explode too close to the ground.

The height (in feet) of a firework t seconds after it is launched is given by $h(t) = 3 + 180t - 16t^2$.

a. The firework is to explode at a height of at least 450 feet. Write an inequality whose solution gives the possible explosion times for the firework.

b. Draw a graph of the height of the firework over time, indicating the times when the firework is at a height of at least 450 feet.

13 The diagram at the right shows an angle in standard position whose terminal side lies in Quadrant II.



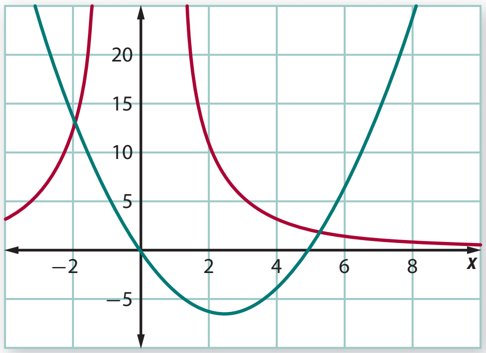
a. Suppose the measure of the angle is θ. Explain why $\cos\theta < 0$, $\sin\theta > 0$, and $\tan\theta < 0$.

b. For each of the following conditions, state the quadrant in which the terminal side of the angle in standard position must lie.

i. $\cos\theta > 0$, $\sin\theta > 0$ **ii.** $\cos\theta > 0$, $\sin\theta < 0$

iii. $\cos\theta < 0$, $\tan\theta > 0$ **iv.** $\sin\theta < 0$, $\tan\theta < 0$

14 The graphs below show the way two different variables depend on x. One function is given by $f(x) = \frac{50}{x^2}$. The other function is given by $g(x) = x^2 - 5x$.



a. Make a copy of these graphs. Identify and label the graphs of $f(x)$ and $g(x)$.

b. What connections between function rules and their graphs allow you to match $f(x)$ and $g(x)$ to their graphs in this case?

c. Color the x-axis of your copy of the graphs to highlight the points corresponding to solutions of the following equation and inequalities. If possible, use the colors suggested.

i. In blue: $\frac{50}{x^2} = x^2 - 5x$

ii. In red: $\frac{50}{x^2} > x^2 - 5x$

iii. In green: $\frac{50}{x^2} < x^2 - 5x$

15 In solving quadratic equations by factoring, you used the fact that if $ab = 0$, then $a = 0$ or $b = 0$. Consider whether similar reasoning can be used to solve a quadratic inequality.

a. Complete a table like this.

If ...	and ... ,	then ... .
$a > 0$	$b > 0$	ab ? 0
$a > 0$	$b < 0$	ab ? 0
$a < 0$	$b > 0$	ab ? 0
$a < 0$	$b < 0$	ab ? 0

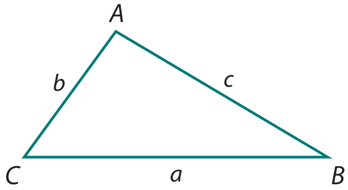
b. Based on your work in Part a, if $ab > 0$, then what can be said about a and b? What if $ab < 0$?

c. For what values of v is the inequality $(v + 3)(v - 4) > 0$ true?

d. How might you expand this kind of reasoning to determine the values of w for which the inequality $(w + 5)(w + 1)(w - 2) < 0$ is true?

16 In the *Reasoning and Proof* unit, you completed an argument to prove one case of the Law of Cosines.

In $\triangle ABC$, $c^2 = a^2 + b^2 - 2ab \cos C$.



Completing the tasks below will help you to see a connection between the Law of Cosines and the Triangle Inequality: $a + b > c$. Recall that $-1 < \cos C < 1$.

a. Explain why it must be the case that $a^2 + b^2 - 2ab \cos C < a^2 + b^2 - 2ab(-1)$.

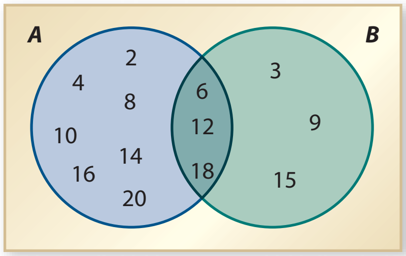
b. Explain why $c^2 < (a + b)^2$.

c. Explain why it follows that $a + b > c$.

d. What form of the Law of Cosines would you use to prove this form of the Triangle Inequality: $a + c > b$?

17 You learned in Investigation 3 that the symbol ∪ indicates the *union* of two sets. The symbol ∩ indicates the *intersection* of two sets.

a. Consider the Venn diagram to the right.



i. How would you describe the numbers in circle A? In circle B?

ii. How would you describe the numbers in $A \cup B$?

iii. Why are the numbers 6, 12, and 18 in $A \cap B$?

b. Consider the integers from -10 to 10. Create Venn diagrams that satisfy the conditions below for C and D. Then describe $C \cup D$ and $C \cap D$ using inequalities.

i. C consists of integers greater than -5. D consists of integers less than 7.

ii. C consists of integers less than -3. D consists of integers greater than 4.

iii. C consists of integers less than 2. D consists of integers less than -1.

Reflections

18 Explain why the values of x indicated by $-3 < x < 7$ are *not* the same as the values indicated by $-3 < x$ or $x < 7$.

19 If $f(x) > c$ only when $a < x < b$, then what are the values of x for which $f(x) \leq c$?

20 In this lesson, the notation (a, b) has been used in two different ways.

- Graph the point $(-3, 2)$ on a coordinate plane.
- Graph the interval $(-3, 2)$ on a number line.

How can you tell, in any particular problem, whether (a, b) refers to a point in the plane or an interval on a number line?

Extensions

21 In your work on problems of this lesson, you focused on inequalities involving linear, quadratic, and inverse variation functions. But inequalities involving other powers and exponential functions are important as well. For each inequality given below:

- sketch a graph of the functions involved.
- use what you know about the functions involved to solve the inequality exactly or by accurate approximation.
- record the solution using symbols and a number line graph.

a. $\sqrt{x} > x \ (x \geq 0)$ **b.** $x^3 < x^2$

c. $2^x < x^2$ **d.** $0.5^x < 0.5x$

22 In the *Trigonometric Methods* unit of Course 2, you studied the patterns of values for the sine, cosine, and tangent functions. In particular, for an angle θ in standard position with point $P(x, y)$ on the terminal side, if $r = \sqrt{x^2 + y^2}$, then $\sin \theta = \frac{y}{r}$, $\cos \theta = \frac{x}{r}$, and $\tan \theta = \frac{y}{x}$, if $x \neq 0$. For each inequality given below:

- use what you know about the functions involved to solve the inequality exactly or by accurate approximation for $0 \leq \theta \leq 180°$.
- record the solution using symbols and a number line graph.

a. $\cos \theta > 0$ **b.** $\cos \theta > \sin \theta$

c. $\tan \theta > 1$ **d.** $\sin \theta < 0.5$

23 For each *compound inequality* below, indicate whether the solution is the union or the intersection of the solutions to the individual inequalities. Then graph the solution of the compound inequality on a number line.

a. $v > -0.5$ and $v \leq 3$ **b.** $x < -2$ or $x \geq 7$

c. $3d + 2 \leq 5$ or $6 - d < 7$ **d.** $2p - 1 < 3$ and $p + 6 \geq 10$

24 Can the solution to an equation ever be an inequality? Can the solution to an inequality ever be an equation? Use sketches of graphs to illustrate your response.

Review

25 For each of these quadratic functions, use the coefficients of the terms to help determine the shape and location of the graph: (1) whether the graph has a maximum or minimum point, (2) the location of the line of symmetry, and (3) where the graph will cross the y-axis.

a. $f(x) = x^2 + 4x - 5$

b. $g(x) = -x^2 - 2x + 8$

c. $h(x) = x^2 - 10x + 25$

d. $j(x) = -0.5x^2 + 2x + 4$

CPMP-Tools

26 Solve each of these quadratic equations in two ways, by factoring and by use of the quadratic formula. Check the solutions you find by substitution in the original equation or by use of a CAS **solve** command.

a. $x^2 - 6x + 5 = 0$

b. $x^2 + 4x = 0$

c. $x^2 - 19 = -3$

d. $2x^2 - x - 6 = 0$

e. $-x^2 - 5x - 4 = 0$

f. $-x^2 + 7x + 6 = 2x$

27 Solve each of the following for x.

a. $3x - 12 < 24$

b. $-2x + 19 > 5x - 2$

c. $10 - 6x < 2x + 90$

d. $x - 3 = \frac{18}{x}$

28 Without using your calculator, sketch a graph of each function below. Then check your sketches using a calculator or computer graphing tool.

a. $f(x) = \frac{1}{x}$

b. $g(x) = x^2$

c. $f(t) = 3(2^t)$

d. $f(p) = -p^2$

e. $h(n) = -\frac{1}{n^2}$

29 Without using the graphing feature of your calculator or computer, find the coordinates of the intersection point for each pair of lines.

a. $x = 5$
$3x + 2y = 24$

b. $3x + 2y = 24$
$y = -0.5x + 6$

c. $y = 8$
$x = 5$

d. $x + y = 200$
$2x + y = 280$

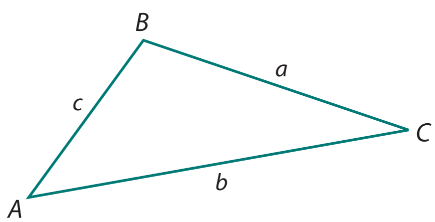
30 You may recall that you can use the Law of Cosines and the Law of Sines to find side lengths and angle measures in any triangle. Below is one form of each of these useful laws for $\triangle ABC$.

Law of Cosines

$c^2 = a^2 + b^2 - 2ab \cos C$

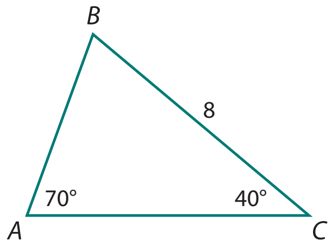
Law of Sines

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

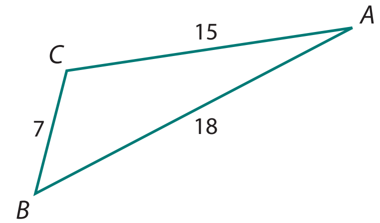


For each triangle below, decide if you should use the Law of Cosines or the Law of Sines to find the indicated side length or angle measure. Then use your choice to find the indicated side length or angle measure.

a. Find AB.



b. Find $m\angle B$.



31 For the linear equations below:

- find coordinates of the y-intercept of the graph.
- find coordinates of the x-intercept of the graph.
- sketch the graph of all solutions.

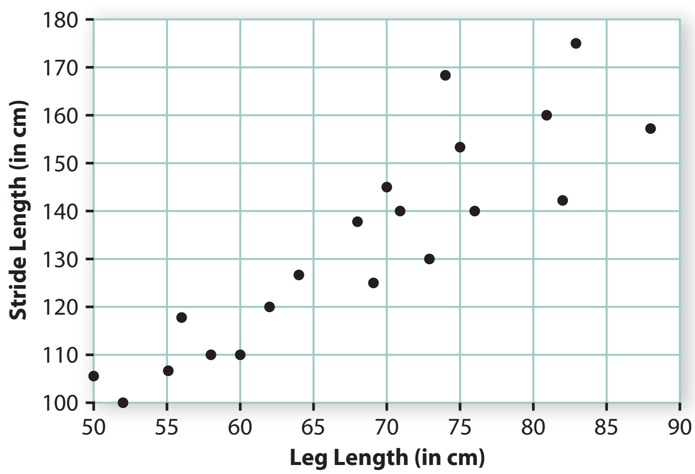
a. $3x + 2y = 6$

b. $8y - 4x = 24$

32 It seems reasonable that the length of a person's stride is related to the length of his or her legs. Manish collected these two measurements for 20 different people.

a. Would you expect the correlation for the data to be positive or negative? Explain your reasoning.

b. Using his data, Manish created the scatterplot below. Describe the direction and strength of the relationship.



c. The linear regression line for these data is $y = 1.78x + 12$. Explain the meaning of the slope of this line in terms of the context.

LESSON 2

Inequalities with Two Variables

Important decisions in business often involve many variables and relations among those variables. The key to making good decisions is finding a way to organize and compare options.

For example, suppose that the manager of an electronics company must plan for and supervise production of two video game systems, the basic CP•I and the advanced CP•II. Assume that demand for both game systems is high, so the company will be able to sell whatever is produced. To plan the work schedule, the manager has to consider the following conditions.

- Assembly of each CP•I model takes 0.6 hours of technician time, and assembly of each CP•II model takes 0.3 hours of technician time. The plant can apply at most 240 hours of technician time to assembly work each day.
- Testing for each CP•I model takes 0.2 hours, and testing of each CP•II model takes 0.4 hours. The plant can apply at most 160 hours of technician time each day for testing.
- Packaging time is the same for each model. The packaging department of the plant can handle at most 500 game systems per day.
- The company makes a profit of \$50 on each CP•I model and \$75 on each CP•II model.

The production planning challenge is to maximize profit while operating under the constraints of limited technician time and packaging time.

Think About This Situation

Suppose that you were the manager of the electronics plant and had to make production plans.

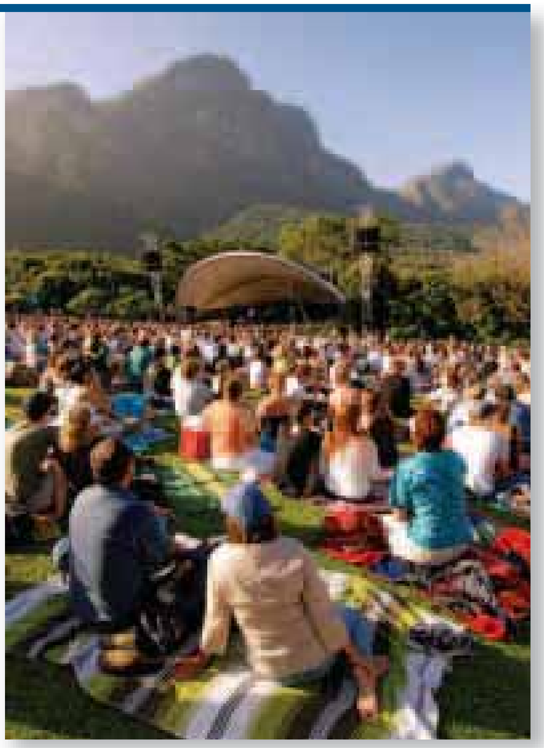
a How would you decide the time estimates for assembly, testing, and packaging?

b How would you decide the expected profit for each game system?

c How might you use all of the given data to decide on the number of CP•I and CP•II models that should be produced to maximize profit for the company?

Many problems like those facing the electronics plant manager are solved by a mathematical strategy called *linear programming*. In this lesson, you will learn how to use this important problem-solving technique and the mathematical ideas and skills on which it depends.

Investigation 1 Solving Inequalities



Many problems that arise in making plans for a business involve functions with two or more independent variables. For example, income for the Old Dominion Music Festival is given by the function $I = 8a + 12g$, where a is the number of admission tickets sold in advance and g is the number of tickets sold at the gate.

If expenses for operating the two-day festival total $2,400, then solutions of $8a + 12g = 2{,}400$ give (a, g) combinations for which festival income will equal operating expenses. But festival organizers are probably interested in earning *more than* $2,400 from ticket sales. They want solutions to the inequality $8a + 12g > 2{,}400$.

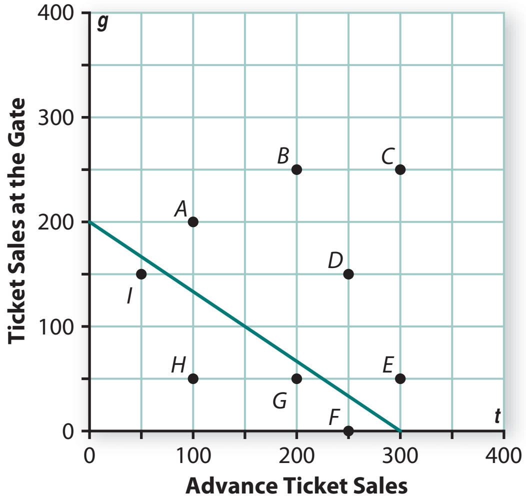
As you work on the problems of this investigation, look for answers to these questions:

How can one find and graph the solutions of a linear inequality in two variables?

How can one find the solutions of a system of inequalities in two variables?

1 The next diagram gives a first quadrant graph of the line representing solutions for the equation $8a + 12g = 2{,}400$, the combinations of tickets sold in advance and at the gate that will give festival income of exactly \$2,400.

Use that graph as a starting point in solving the inequality $8a + 12g > 2{,}400$.



a. For each point on the diagram, give the (a, g) coordinates and the festival income from that combination of tickets sold in advance and tickets sold at the gate.

b. Based on your work in Part a, describe the graphical representation of solutions to these inequalities.

  i. $8a + 12g > 2{,}400$

  ii. $8a + 12g < 2{,}400$

c. Solve the equation $8a + 12g = 2{,}400$ for g in terms of a.

d. Use what you know about manipulation of inequality statements to express each inequality in Part b in an equivalent form $g < \ldots$ or $g > \ldots$. Then explain how those equivalent inequalities help to describe the solution graphs for the original inequalities.

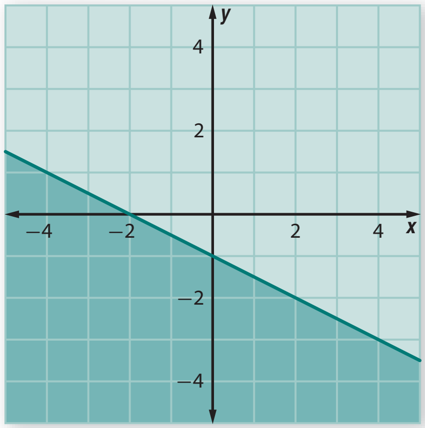
2 A local bank is giving away souvenir Frisbees and sun visors at the Old Dominion Music Festival. The Frisbees will cost the bank \$4 each, and the visors will cost the bank \$2.50 each. The promotional cost for the bank depends on the number of Frisbees F and the number of visors V given away at the festival. The bank has budgeted \$1,000 for the purchase of these items.

a. Write an inequality that represents the question, "How many souvenir Frisbees and visors can the bank give away for a total cost of no more than \$1,000?"

b. Draw a graph that shows the region of the first quadrant in the coordinate plane containing all points (F, V) that satisfy the inequality in Part a.

Graphing Conventions In Problems 1 and 2, the only meaningful solutions of inequalities were in the first quadrant. Negative values of a, g, F, and V make no sense in those situations. However, the ideas and graphing techniques you developed in work on those problems can be applied to situations without the constraint of only positive values.

For example, the following graph shows solutions with both positive and negative values for $x + 2y < -2$ and $x + 2y > -2$.



3 Compare the inequalities and the indicated graph regions.

a. Which region corresponds to solutions of $x + 2y < -2$? How did you decide?

b. How would you graph the solutions to a linear inequality like $x - y > 3$?

Just as with inequalities in one variable, the solution region of an inequality in two variables is generally bounded by the solution of a corresponding equation. Sometimes the boundary is *included* in the solution of the inequality, indicated on a graph by a *solid* boundary. A *dashed* boundary on a graph indicates that points on the boundary are *excluded* from the solution.

4 Draw graphs that show all points with x- and y-coordinates between −10 and 10 and satisfying these linear inequalities in two variables. Use either solid or dashed boundary lines as appropriate.

a. $3x - 2y < 12$

b. $2x + y \geq 4$

c. $8x - 5y > 20$

d. $4x + 3y \leq 15$

5 Suppose that the organizers of the Old Dominion Music Festival can sell no more than 1,000 admission tickets due to space constraints.

a. Write an inequality whose solutions are the (a, g) pairs for which the number of tickets sold will be no more than 1,000.

b. Draw a graph that uses appropriate boundary lines to show the region of the first quadrant in the coordinate plane that contains all points (a, g) satisfying both the inequality $8a + 12g > 2{,}400$ (from Problem 1) and the inequality from Part a of this problem.

6 Suppose that the bank wants to give away at least 300 promotional items at the Old Dominion Music Festival.

a. Write an inequality whose solutions are the (F, V) combinations for which the total number of promotional items will be 300 or more.

b. Draw a graph that shows the region of the first quadrant in the coordinate plane containing all points (F, V) that satisfy both the inequality $4F + 2.5V \leq 1{,}000$ (from Problem 2) and the inequality from Part a of this problem.

You have solved systems of equations in previous work. Recall that the goal in solving a system of equations is to find values of the variables that satisfy all equations in the system. Similarly, the goal in solving a *system of inequalities* is to find values of the variables that satisfy all inequalities in the system. As with systems of equations, systems of inequalities can be solved by graphing the solution of each inequality in the system and finding the points of intersection as you did in Problems 5 and 6.

7 Draw graphs that show the solutions of these systems of inequalities.

a. $\begin{cases} 2x - 3y > -12 \\ x + y \geq -2 \end{cases}$

b. $\begin{cases} 3x - 4y > 18 \\ 5x + 2y \leq 15 \end{cases}$

c. $\begin{cases} y > 4 - x^2 \\ 2x - y > -3 \end{cases}$

d. $\begin{cases} y > x^2 - 4x - 5 \\ 2x - y \geq -2 \end{cases}$

Summarize the Mathematics

In this investigation, you developed strategies for graphing the solution of a linear inequality in two variables and for graphing the solution of a system of inequalities in two variables.

a Describe a general strategy for graphing the solution of a linear inequality in two variables.

b How do you show whether the points on the graph of the corresponding linear equation are included as solutions?

c Describe the goal in solving a system of inequalities and a general strategy for finding the solutions.

Be prepared to share your ideas and reasoning with the class.

Check Your Understanding

Jess has made a commitment to exercise in order to lose weight and improve overall fitness. Jess would like to burn *at least* 2,000 calories per week through exercise but wants to schedule *at most* five hours of exercise each week.

Jess will do both walking and bike riding for exercise. Brisk walking burns about 345 calories per hour, and biking at a moderate pace burns about 690 calories per hour.

a. Write a system of linear inequalities that describes Jess' exercise goals.

b. Identify at least three (*number of hours walking, number of hours biking*) combinations that satisfy both of Jess' conditions.

c. Graph the set of all points that satisfy the system from Part a.

Investigation 2 Linear Programming—A Graphic Approach

Linear programming problems, like that faced by managers of the video game factory described at the start of this lesson, involve finding an optimum choice among many options. As you work on the problems in this investigation, look for an answer to this question:

How can coordinate graphs be used to display and analyze the options in linear programming decision problems?

Production Planning The production-scheduling problem at the electronics plant requires the plant manager to find a combination of CP•I and CP•II models that will give greatest profit. But there are **constraints** or limits on the choice. Each day, the plant has capacity for:

- *at most* 240 hours of assembly labor with each CP•I requiring 0.6 hours and each CP•II requiring 0.3 hours.
- *at most* 160 hours of testing labor with each CP•I requiring 0.2 hours and each CP•II requiring 0.4 hours.
- packing *at most* 500 video game systems with each model requiring the same time.

The company makes profit of \$50 on each CP•I model and \$75 on each CP•II model.

1 One way to search for the production plan that will maximize profit is to make some guesses and test the profit prospects for those guesses. Here are three possible production plans for CP•I and CP•II video game models.

Plan 1: Make 100 of model CP•I and 200 of model CP•II.
Plan 2: Make 200 of model CP•I and 100 of model CP•II.
Plan 3: Make 400 of model CP•I and 100 of model CP•II.

a. Check each production plan by answering the following questions.

i. Will the required assembly time be within the limit of 240 hours per day?

ii. Will the required testing time be within the limit of 160 hours per day?

iii. Will the number of systems produced fall within the packing limit of 500 units per day?

iv. If the constraints are satisfied, what profit will be earned?

b. Design and check a plan of your own that you think will produce greater profit than Plans 1, 2, and 3 while satisfying the assembly, testing, and packing constraints.

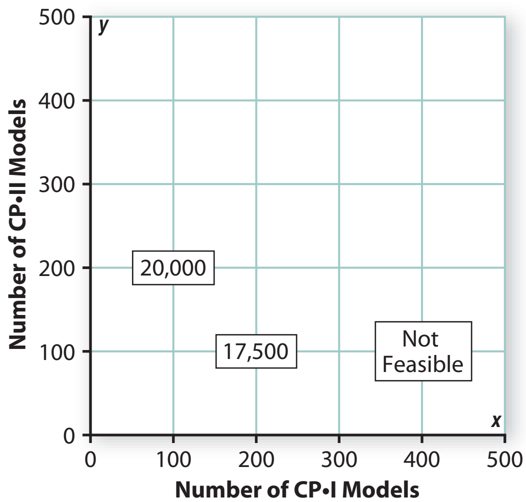
As you compared the possible production plans in Problem 1, you checked several different constraints and evaluated the profit prospects for each combination of CP•I and CP•II video game systems. However, you checked only a few of many possible production possibilities.

It would be nice to have a systematic way of organizing the search for a maximum profit plan. One strategy for solving **linear programming** problems begins with graphing the options. If x represents the number of CP•I game systems produced and y represents the number of CP•II game systems produced, then the scheduling goal or *objective* is to find a pair (x, y) that meets the constraints and gives maximum profit.

Using a grid like the one below, you can search for the combination of CP•I and CP•II video game system numbers that will give maximum profit. The point (100, 200) represents production of 100 CP•I and 200 CP•II game systems. These numbers satisfy the constraints and give a profit of

$$\$50(100) + \$75(200) = \$20{,}000.$$

Video Game System Profits (in dollars)



2 Each **lattice point** on the grid (where horizontal and vertical grid lines intersect) represents a possible combination of CP•I and CP•II video game system numbers. Points with coordinates satisfying all the constraints are called **feasible points**.

a. What do the labels on the points with coordinates (200, 100) and (400, 100) tell about the production planning options?

b. Collaborate with others to check the remaining lattice points on the given graph to see which are feasible points and which are not. For each feasible point, find the profit for that production plan and record it on a copy of the graph. Label each nonfeasible point "NF."

c. Based on the completed graph:

i. describe the region where coordinates of lattice points satisfy all three constraints.

ii. pick the combination of CP•I and CP•II video game systems that you think the factory should produce in order to maximize daily profit. Be prepared to explain why you believe your answer is correct.

Balancing Astronaut Diets Problems like the challenge of planning video game system production to maximize profit occur in many quite different situations. For example, think about the variables, constraints, and objectives in choosing the foods you eat. Usually, you choose things that taste good. But it is also important to consider the cost of the food and the ways that it satisfies your body's dietary needs.

3 For some people, athletes and astronauts in particular, selection of a good diet is a carefully planned scientific process. Each person wants a high-performance diet at minimal cost. In the case of astronauts, the goal might be minimal total food weight onboard the spacecraft.

Consider the following simplified version of the problem facing NASA flight planners who must provide food for astronauts.

Suppose there are two kinds of food to be carried on a space shuttle trip, special food bars and cartons of a special drink.

- Each food bar provides 5 grams of fat, 40 grams of carbohydrate, and 8 grams of protein.
- Each drink carton provides 6 grams of fat, 25 grams of carbohydrate, and 15 grams of protein.
- Minimum daily requirements for each astronaut are *at least* 61 grams of fat, *at least* 350 grams of carbohydrate, and *at least* 103 grams of protein.
- Each food bar weighs 65 grams, and each drink weighs 118 grams.

The goal is to find a combination of food bars and drinks that will fulfill daily requirements of fat, carbohydrate, and protein with minimum total weight onboard the spacecraft.

This probably seems like a complicated problem. But you can get a good start toward a solution by doing some systematic testing of options.

a. For each of these numbers of food bars and drink cartons, check to see if they provide at least the daily minimums of fat, carbohydrate, and protein. Then find the total weight of each feasible combination.

i. 4 food bars and 10 cartons of drink

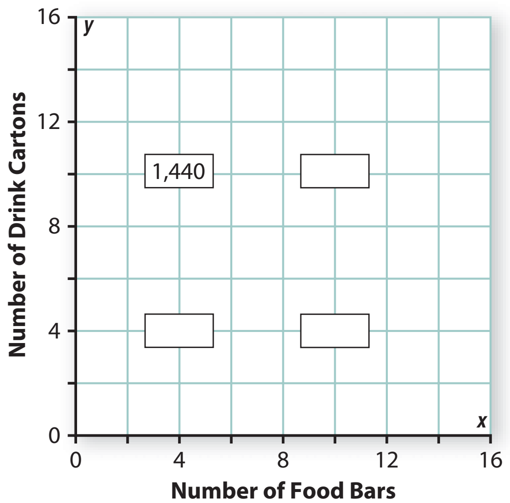
ii. 10 food bars and 4 cartons of drink

iii. 4 food bars and 4 cartons of drink

iv. 10 food bars and 10 cartons of drink

b. Record your findings on a copy of the following graph. The case of 4 food bars and 10 drink cartons has been plotted already.

Food and Drink Weights (in grams)



c. Collaborate with others to test the remaining lattice points on the grid to get a picture of the **feasible region**, the points with coordinates that meet all constraints. For each feasible point, find the total weight of the food and drink. Plot it on a copy of the grid.

4 Now analyze the pattern of *feasible points* and *objective values* (weights) for possible astronaut diets.

a. Describe the shape of the feasible region.

b. Study the pattern of weights for points in the feasible region. Decide on a combination of food bars and drink cartons that you think will meet the diet constraints with minimum weight. Be prepared to explain why you believe your answer is correct.

Summarize the Mathematics

Problems that can be solved by linear programming have several common features: variables, constraints, and an objective.

a What are the variables, constraints, and objective in the video game system production problem?

b What are the variables, constraints, and objective in the astronaut diet problem?

c What are *feasible points* and the feasible region in a linear programming problem?

d What do coordinates of the feasible points and the "not feasible" points tell you in the video game system production problem? In the astronaut diet problem?

Be prepared to explain your ideas to the class.

Check Your Understanding

Suppose that a new store plans to lease selling space in a mall. The leased space would be divided into two sections, one for books and the other for music and videos.

- The store owners can lease up to 10,000 square feet of floor space.
- Furnishings for the two kinds of selling space cost $5 per square foot for books and $7 per square foot for music and videos. The store has a budget of at most $60,000 to spend for furnishings.
- Each square foot of book-selling space will generate an average of $75 per month in sales. Each square foot of music- and video-selling space will generate an average of $95 per month in sales.

The store owners have to decide how to allocate space to the two kinds of items, books or music/video, to maximize monthly sales.

a. Identify the variables, constraints, and objective in this situation.

b. Find three feasible points and one point that is not feasible.

c. Evaluate the predicted total monthly sales at the three feasible points.

d. Plot the three feasible points and the nonfeasible point on an appropriate grid and label each feasible point with the corresponding value of predicted total monthly sales.

e. Which of the three sample feasible points comes closest to the problem objective?

Investigation 3 Linear Programming—Algebraic Methods

As you worked on the production-planning problem for the video game factory and the astronaut diet problem, you probably thought, "There's got to be an easier way than testing all those possible combinations." Computers have been programmed to help explore the options. But to use those tools, it is essential to express the problem in algebraic form. In this investigation, look for answers to these questions:

How can constraints and objectives of linear programming problems be expressed in symbolic form?

How can algebraic and graphical methods be combined to help solve the problems?

Video Game System Production Revisited Think again about the objective and the constraints in the production-planning problem. Each CP•I video game system earns \$50 profit, and each CP•II system earns \$75 profit. But daily production is constrained by the times required for assembly, testing, and packaging.

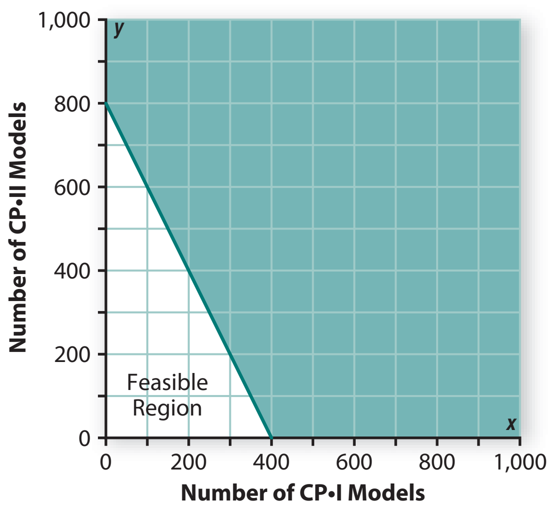
1 If x represents the number of CP•I models and y represents the number of CP•II models produced in a day, what algebraic rule shows how to calculate total profit P for the day? This rule is called the **objective function** for the linear programming problem because it shows how the goal of the problem is a function of, or depends on, the independent variables x and y.

2 Recall that assembly time required for each CP•I model is 0.6 hours, and assembly time required for each CP•II model is 0.3 hours. Assembly capacity is limited by the constraint that the factory can use at most 240 hours of technician time per day.

a. Explain how the linear inequality $0.6x + 0.3y \leq 240$ represents the assembly time constraint.

b. The graph below shows points meeting the assembly time constraint. *Points that are **not** feasible have been shaded out of the picture.*

Video Game System Assembly Time (in hours)



i. Why does the graph only show points in the first quadrant?

ii. Which feasible point(s) do you think will lead to greatest daily profit for the company? Test your ideas by evaluating the objective function at a variety of feasible points.

3 Next, recall that testing of each CP•I model requires 0.2 hours of technician time and testing of each CP•II model requires 0.4 hours. The factory can apply at most 160 hours of technician time to testing each day.

a. Write an inequality that expresses the testing time constraint. Be prepared to explain how you know that your algebraic representation of the constraint is correct.

b. Graph the solutions of that inequality, shading *out* the region of nonfeasible points.

c. Which feasible point(s) do you think will lead to greatest daily profit for the factory? Test your ideas by evaluating the objective function at a variety of feasible points.

4 Finally, recall that the factory can package and ship at most 500 video game systems each day.

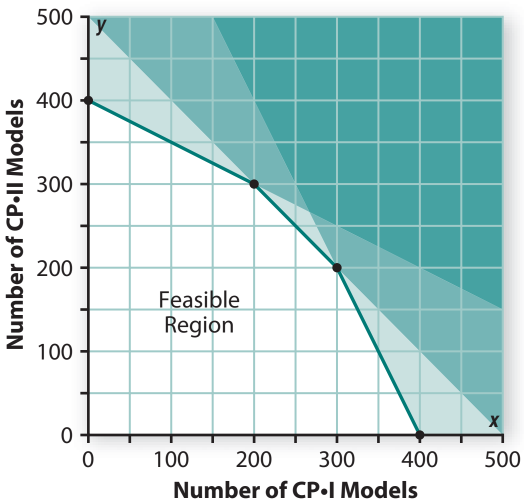
a. Write an inequality that expresses the packaging/shipping constraint. Be prepared to explain how you know that your algebraic expression of the constraint is correct.

b. Graph the solutions of that inequality, shading *out* the region of nonfeasible points.

c. Which feasible point(s) do you think will lead to greatest daily profit for the factory? Test your ideas.

5 In work on Problems 2–4, you developed some ideas about how to maximize profit while satisfying each problem constraint separately. The actual production-planning problem requires maximizing profit while satisfying all three constraints. The feasible region for this problem is shown as the unshaded region in the diagram below.

Video Game System Production Feasibility



a. On a copy of the graph, use results from Part a of Problems 2, 3, and 4 to label each segment of the feasible region boundary with its corresponding linear equation.

b. Use the graph to estimate coordinates of points at the corners of the feasible region. Then explain how you could check the accuracy of your estimates and how you could calculate the coordinates by algebraic methods.

c. Now think about the objective function $P = 50x + 75y$.

i. How does the value of P change as the numbers x and y increase within the feasible region?

ii. Where in the feasible region would you expect a combination of production numbers (x, y) to give maximum profit?

iii. List coordinates of some likely maximum profit points. Evaluate the profit corresponding to each point.

Video Game System Profit

Number of CP•I Models	Number of CP•II Models	Profit (in dollars)

iv. Of these production options, which gives the maximum profit?

d. Compare your choice of maximum profit production plan to those of others in your class and resolve differences.

Astronaut Diet Planning Revisited Think again about the objective and constraints in the astronaut diet-planning problem. Each food bar weighs 65 grams, and each drink carton weighs 118 grams. But the diet plan is constrained by the minimum daily requirements of fat, carbohydrate, and protein.

6 If f represents the number of food bars and d represents the number of drink cartons in a daily diet, what algebraic rule shows how to calculate total weight W of that food? (This rule is the objective function for the diet linear programming problem.)

7 Recall that each food bar provides 5 grams of fat, 40 grams of carbohydrate, and 8 grams of protein. Each drink carton provides 6 grams of fat, 25 grams of carbohydrate, and 15 grams of protein. Write inequalities that express the constraints of providing the daily astronaut diet with:

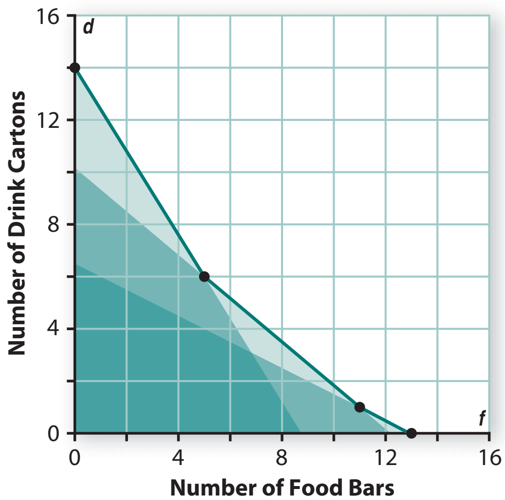
a. at least 61 grams of fat.

b. at least 350 grams of carbohydrate.

c. at least 103 grams of protein.

8 The next graph shows the feasible region for the astronaut diet problem. The feasible region for this problem is shown as the unshaded region in the graph.

Astronaut Food and Drink Feasibility



a. On a copy of the graph, label each segment of the feasible region boundary with its corresponding linear equation.

b. Use the graph to estimate coordinates of points at the corners of the feasible region. Then check the accuracy of your estimates by algebraic methods.

c. Now think about the objective function $W = 65f + 118d$.

i. How does the value of W change as the numbers f and d decrease within the feasible region?

ii. Where in the feasible region do you expect a pair of numbers (f, d) to give minimum weight?

iii. List coordinates of some likely minimum weight points. Evaluate the weight corresponding to each point.

Astronaut Diet Options

Number of Food Bars	Number of Drink Cartons	Weight (in grams)

iv. Of these diet options, which gives minimum weight?

d. Compare your choice of minimum weight diet plan to those of others in your class and resolve differences.

Summarize the Mathematics

In this investigation, you explored ways that reasoning with symbolic expressions, inequalities, and graphs helps to solve linear programming problems.

a) Why are the constraints in the video game system production and astronaut diet planning problems most accurately expressed as inequalities rather than equations?

b) What shapes would you expect for the feasible regions in other linear programming problems where the goal is to maximize some objective function? In problems where the goal is to minimize some objective function?

c) What seems to be the best way to locate feasible points that maximize (or minimize) the objective function in a linear programming problem?

Be prepared to explain your ideas to the class.

Check Your Understanding

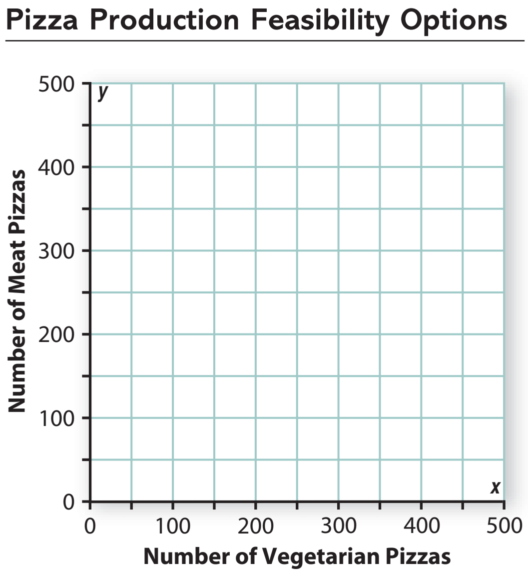
Paisan's Pizza makes gourmet frozen pizzas to sell to supermarket chains. The company makes two deluxe pizzas, one vegetarian and the other with meat.

- Each vegetarian pizza requires 12 minutes of labor, and each meat pizza requires 6 minutes of labor. The plant has at most 3,600 minutes of labor available each day.
- The plant freezer can handle at most 500 pizzas per day.
- Vegetarian pizza is not quite as popular as meat pizza, so the company can sell at most 200 vegetarian pizzas each day.
- Sale of each vegetarian pizza earns Paisan's \$3 profit. Each meat pizza earns \$2 profit.

Paisan's Pizza would like to plan production to maximize daily profit.

a. Translate the objective and constraints in this situation into symbolic expressions and inequalities.

b. On a copy of a grid like the one below, graph the system of constraints to determine the feasible region for Paisan's linear programming problem. Label each segment of the feasible region boundary with the linear equation that determines it.



c. Evaluate the objective function at the point(s) that you believe will yield greatest daily profit for Paisan's Pizza. Compare that profit figure to the profit for several other feasible points that you think might be "next best."

On Your Own

Applications

1 Match each inequality listed in Parts a–j with the region I–X that shows points (x, y) that satisfy the inequality. The scales on the coordinate axes are 1.

a. $y \geq x$

b. $y \leq x$

c. $2x + 3y \leq 3$

d. $2x + 3y \geq 3$

e. $y \geq 0.5x - 2$

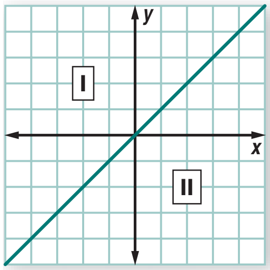
f. $y \leq 0.5x - 2$

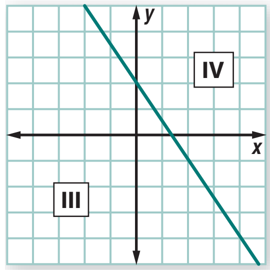
g. $x \leq 2$

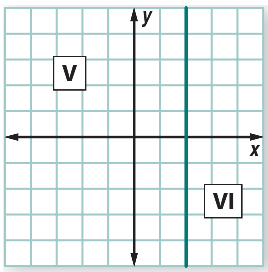
h. $x \geq 2$

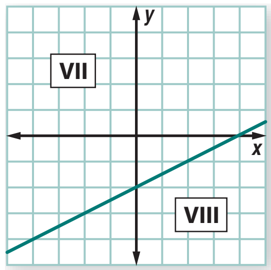
i. $3x + 2y \geq 4$

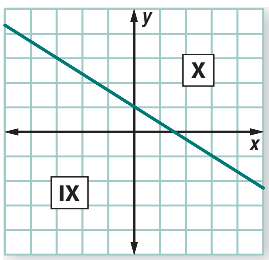
j. $3x + 2y \leq 4$











2 Graph the solution of $5x - 2y > 10$.

3 The symbol of Columbia, Maryland, is a people tree that stands by Lake Kittamaqundi. The tree is 14 feet tall with 66 gilded people as branches. As part of a renovation of downtown Columbia in 1992, residents purchased engraved brick pavers to pay for regilding of the people tree. The pavers were used to cover a new plaza around the tree.

Brick pavers engraved with one line of text sold for \$25, and pavers engraved with two lines of text sold for \$30. Each brick cost \$18 to buy, engrave, and install. Regilding the people tree sculpture cost \$11,000.

a. Write an inequality whose solution gives the combinations of *one-line pavers* n_1 and *two-line pavers* n_2 that would cover the cost of regilding the people tree sculpture.

b. Draw a graph that uses shading to show the region of the coordinate plane containing all points (n_1, n_2) that satisfy the inequality.

4 Refer back to Applications Task 3. Suppose that the Columbia people tree plaza has space for 1,200 engraved pavers.

a. Write a system of inequalities whose solution will give the (n_1, n_2) combinations for which space is available and the cost of regilding the people tree sculpture will be covered.

b. Draw a graph that shows the region of the first quadrant in the coordinate plane that contains all points (n_1, n_2) satisfying the system of inequalities from Part a.

5 Graph the solutions of the following systems of inequalities in a coordinate plane.

a. $\begin{cases} x - y < 7 \\ 3x + 2y > 9 \end{cases}$

b. $\begin{cases} y \geq x^2 + 5x \\ 3x + y \leq 15 \end{cases}$

6 The Bestform Ring Company makes class rings for high schools and colleges all over the country. The rings are made in production runs that each yield 100 rings. Each production run is a three-step process involving molding, engraving, and polishing.

The following chart gives information about the time required for the steps in each production run of high school or college rings and the time that machines and operators are available during one day.

Class Ring Production

Stage in Ring Making	Time to Make 100 High School Rings (in hours)	Time to Make 100 College Rings (in hours)	Machine and Operator Time Available Each Day (in hours)
Molding	1.2	2	14
Engraving	0.6	3	15
Polishing	2.0	2	20

a. Test the feasibility of these possible daily production plans.

Plan 1: 2 production runs of high school rings and 3 production runs of college rings

Plan 2: 3 production runs of high school rings and 5 production runs of college rings

Plan 3: 7 production runs of high school rings and 1 production run of college rings

Plan 4: 4 production runs of high school rings and 4 production runs of college rings

Plan 5: 5 production runs of high school rings and 3 production runs of college rings

b. The company makes \$500 profit on each production run of high school rings and \$525 profit on each production run of college rings.

i. Compute the daily profit from each feasible production plan in Part a.

ii. See if you can find a feasible production plan that results in higher profit than any of those in Part a.

7 The Junior Class of Oakland Mills High School sells drinks at the Columbia Fair to raise funds for the Junior Prom. The juniors mix and sell two drinks, Carnival Juice and Lemon Punch.

- Each batch of Carnival Juice uses two liters of orange juice and two liters of lemonade.
- Each batch of Lemon Punch uses one liter of orange juice and three liters of lemonade.
- The students have 120 liters of orange juice and 200 liters of lemonade to use.
- The profit is \$9 per batch on the Carnival Juice and \$12 per batch on the Lemon Punch.

a. Test the feasibility of these plans for mixing Carnival Juice and Lemon Punch.

Plan 1: 40 batches of Carnival Juice and 20 batches of Lemon Punch

Plan 2: 30 batches of Carnival Juice and 30 batches of Lemon Punch

Plan 3: 40 batches of Carnival Juice and 50 batches of Lemon Punch

Plan 4: 50 batches of Carnival Juice and 20 batches of Lemon Punch

b. Find the profit that the Junior Class will earn from each feasible combination of batches of Carnival Juice and Lemon Punch in Part a.

c. See if you can you find a feasible combination of drink batches that results in higher profit than any of those in Part a.

8 When candidates for political office plan their campaigns, they have choices to make about spending for advertising. Suppose that advisers told one candidate that radio and television ads might reach different audiences in the following ways.

- Every dollar spent on radio advertising will assure that the candidate's message will reach 5 Democratic voters, 2 Republican voters, and 4 Independent voters.
- Every dollar spent on television advertising will assure that the candidate's message will reach 4 Democratic voters, 4 Republican voters, and 1 Independent voter.

The candidate's goals are to reach at least 20,000 Democratic voters, 12,000 Republican voters, and 8,000 Independent voters with a minimum total advertising expense.

a. Test the feasibility of these campaign spending plans for reaching voters.

Plan 1: \$2,000 on radio advertising and \$3,000 on television advertising

Plan 2: \$3,000 on radio advertising and \$2,000 on television advertising

Plan 3: \$2,000 on radio advertising and \$2,000 on television advertising

Plan 4: \$4,000 on radio advertising and \$3,000 on television advertising

b. **i.** Find the total advertising cost for each feasible campaign plan in Part a.

ii. See if you can find a feasible combination of advertising dollars that results in a lower total expense for the candidate than any of those in Part a.

9 Refer back to Applications Task 6. Use *h* to represent the number of production runs of high school rings and *c* to represent the number of production runs of college rings.

a. Write inequalities or rules that represent:

i. the constraint on time available for molding rings.

ii. the constraint on time available for engraving rings.

iii. the constraint on time available for polishing rings.

iv. the objective function.

b. Use the constraint inequalities you wrote in Part a to create a graph showing the feasible region of the ring production planning problem. Label each segment of the feasible region boundary with the equation of that line.

c. Find the production plan that will maximize profit for the Bestform Ring Company. Be prepared to explain how you know that your answer is correct.

10 Refer back to Applications Task 7. Use *C* to represent the number of batches of Carnival Juice and *L* to represent the number of batches of Lemon Punch to be mixed.

a. Write inequalities or expressions that represent:

i. the constraint on amount of orange juice available.

ii. the constraint on amount of lemonade available.

iii. the objective function.

b. Use the constraint inequalities you wrote in Part a to create a graph showing the feasible region of the juice sale problem. Label each segment of the feasible region boundary with the equation of that line.

c. Find the plan that will maximize profit for the Junior Class from drink sales. Be prepared to explain how you know that your answer is correct.

11 Refer back to Applications Task 8. Use *r* to represent the number of dollars spent on radio advertising and *t* to represent the number of dollars spent on television advertising.

a. Write symbolic expressions that represent:

i. the constraint on number of Democratic voters to be reached.

ii. the constraint on number of Republican voters to be reached.

iii. the constraint on number of Independent voters to be reached.

iv. the objective function.

b. Use the constraint inequalities you wrote in Part a to create a graph showing the feasible region of the campaign message problem. Label each segment of the boundary with the equation of that line.

c. Use the results of your work on Parts a and b to find the plan that will minimize total advertising expense. Be prepared to explain how you know that your answer is correct.

12 Sketch the feasible regions defined by the following inequalities. Use the given equations for profit P and cost C to find (x, y) combinations yielding maximum profit or minimum cost within the feasible regions.

a. $3y - 2x \geq 6$

$0 \leq x \leq 4$

$y \leq 5$

$P = 5x + 3y$

b. $x \leq 10$

$2x + y \geq 20$

$y \leq 14$

$C = 20x + 5y$

13 The director of the Backstage Dance Studio must plan for and operate many different classes, 7 days a week, at all hours of the day.

Each Saturday class fills up quickly. To plan the Saturday schedule, the director has to consider the following facts.

- It is difficult to find enough good teachers, so the studio can offer at most 8 tap classes and at most 5 jazz classes.
- Limited classroom space means the studio can offer at most 10 classes for the day.
- The studio makes profit of \$150 from each tap class and \$250 from each jazz class.

a. Write and graph the constraint inequalities.

b. The director wants to maximize profit. Write the objective function for this situation.

c. Find the schedule of classes that gives the maximum profit.

d. The director of Backstage Dance Studio really wants to promote interest in dance, so she also wants to maximize the number of children who can take classes. Each tap class can accommodate 10 students, and each jazz class can accommodate 15 students. Find the schedule that gives maximum student participation.

14 A city recreation department offers Saturday gymnastics classes for beginning and advanced students. Each beginner class enrolls 15 students, and each advanced class enrolls 10 students. Available teachers, space, and time lead to the following constraints.

- There can be at most 9 beginner classes and at most 6 advanced classes.
- The total number of classes can be at most 7.
- The number of beginner classes should be at most twice the number of advanced classes.

a. What are the variables in this situation?

b. Write algebraic inequalities giving the constraints on the variables.

c. The director wants as many children as possible to participate. Write the objective function for this situation.

d. Graph the constraints and outline the feasible region for the situation.

e. Find the combination of beginner and advanced classes that will give the most children a chance to participate.

f. Suppose the recreation department director sets new constraints for the schedule of gymnastics classes.

- The same limits exist for teachers, so there can be at most 9 beginner and 6 advanced classes.
- The program should serve at least 150 students with 15 in each beginner class and 10 in each advanced class.

The new goal is to minimize the cost of the program. Each beginner class costs \$500 to operate, and each advanced class costs \$300. What combination of beginner and advanced classes should be offered to achieve the objective?

Connections

15 Identify the geometric shapes with interiors defined by these systems of inequalities.

a. $x \leq 1$, $y \leq 1$, $x \geq -1$, and $y \geq -1$

b. $-2x + y \leq 2$, $2x + y \geq -2$, $2x + y \leq 2$, and $-2x + y \geq -2$

c. $y \leq x$, $y \leq 1$, $y \geq x - 2$, and $y \geq 0$

d. $x + y \leq 1$, $x - y \leq 1$, $-x + y \geq 1$, and $x + y \geq 1$

16 Identify the geometric shapes with interiors defined by these systems of inequalities.

a. $x^2 + y^2 \leq 25$, $x \geq 0$, and $y \leq 0$

b. $x^2 + y^2 \leq 25$ and $y \geq x$

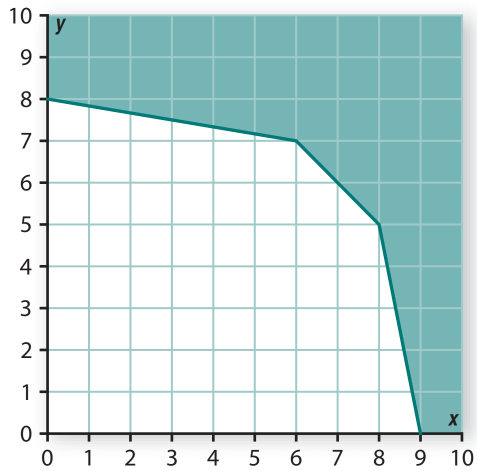
17 The linear programming problems in this lesson involve maximizing or minimizing a linear objective function in two variables. In previous work, you sought to find the maximum value of a quadratic function in one variable. For example, when Miguel Tejada hit a homerun in the 2005 major league baseball All-Star game, the function $h(t) = -16t^2 + 64t + 3$ might have been a reasonable model for the relationship between height of the ball (in feet) and time (in seconds) after it left his bat.

a. At what time was his hit at its maximum height and what was that height?

b. At what height did his bat hit the ball?

c. If the ball reached the outfield seats at a point 20 feet above the playing field, what was the approximate time it landed there?

18 Suppose that the following diagram shows the feasible region for a linear programming problem and that the objective function is $P = 2x + 3y$.



a. On a copy of the diagram, plot the line $2x + 3y = 6$. Explain why the objective function has the value 6 at each point on that line.

b. On the same diagram, plot the line $2x + 3y = 12$. What is the value of the objective function for each point on that line?

c. On the same diagram, plot the line $2x + 3y = 21$. What is the value of the objective function for each point on that line?

d. On the same diagram, plot the line $2x + 3y = 33$. What is the value of the objective function for each point on that line?

e. Explain how the pattern of results in Parts a–d suggests that the lines with equation $2x + 3y = k$ contain no feasible points if $k > 33$.

f. Explain how your work on Parts a–e illustrates the fact that the maximum objective function value in a linear programming problem like this will always occur at a "corner" of the feasible region.

19 The assembly and packaging of CP•I and CP•II video game systems is only one part of the business activity needed to develop a new product. The table below shows some other key tasks, their times to completion, and other tasks on which they depend. Use the information to find the *minimal time to completion* (earliest finish time) for the whole venture of designing and bringing to market products like the CP•I and CP•II video game systems.

Task	Time to Completion	Prerequisites
T1: Design of Game Concept	6 months	
T2: Market Research	3 months	T1
T3: Game Programming	3 months	T1
T4: Design of Production	2 months	T3
T5: Advance Advertising	4 months	T2
T6: Production and Testing of Initial Units	1 month	T4

Reflections

20 How do the solutions of the following two inequalities differ?

$2x + y \leq 6$ $\qquad\qquad$ $2x + y < 6$

How could you show these differences in sketches of their graphs?

21 Solving linear programming problems includes finding the boundary of the feasible region. Describe at least three different ways to find the points where the boundary lines defining the region intersect.

22 How are the goals and constraints of linear programming problems similar to other optimization problems like finding a minimal spanning tree in a graph, analyzing a PERT chart, or finding selling prices that maximize profit from sale of a product?

23 Look back at your solutions of the linear programming problems in this lesson.

a. What kind of values made sense for the variables in these problems?

b. How would you proceed if you determine the optimal solution to a linear programming problem occurs at a point that has noninteger coordinates?

24 Why do you think linear programming has the word *linear* in its name? Do you think the process would work to find optimal solutions if any of the constraints or the objective function were not linear? Why or why not?

25 Realistic linear programming problems in business and engineering usually involve many variables and constraints. Why do you think that linear programming was not used in business and engineering until fairly recently?

Extensions

26 Regular cell phone use involves a combination of talk and standby time. CellStar claims that its cell phone will get up to 200 minutes of talk time or up to 5 days (7,200 minutes) of standby time for a single battery charge. Consider the possible combinations of talk time and standby time before the battery would need to be recharged.

a. Write an inequality whose solution will give the (t, s) combinations that will not completely drain a battery that begins use fully charged.

b. Draw a graph that uses appropriate boundary lines to show the region of the coordinate plane that contains all points with coordinates (t, s) that satisfy the inequality in Part a.

c. Solve the inequality from Part a for s to show how many minutes of standby time remain after t minutes of talk time before the battery will need to be recharged.

27 Graph the solution of $\frac{x}{y} > 1$.

28 Look back at the mall space leasing problem in the Check Your Understanding on page 136. Suppose a new survey of consumer interests indicates that each square foot of book-selling space will generate an average of \$65 per month in sales and each square foot of music- and video-selling space will generate an average of \$95 per month in sales. What space allocations would you recommend to the store owners in this case? Explain your reasoning.

CPMP-Tools

29 Refer back to Extensions Task 26. Suppose that Jovan has a CellStar phone and is planning to take a bus from Durham, North Carolina, to San Antonio, Texas, to visit his aunt. The trip is scheduled to take 1 day, 8 hours, and 10 minutes. Jovan will most likely not be able to recharge the phone during the long bus trip and would like to make sure that the phone maintains a charge during the entire trip.

a. Jovan decides that the solution to the following system will give the number of minutes t of talk time and s of standby time for which the cell phone will maintain a charge for the whole trip.

$$\begin{cases} \frac{t}{200} + \frac{s}{7{,}200} < 1 \\ t + s > 1{,}930 \end{cases}$$

Do you agree with Jovan? Why or why not?

b. Graph the solution to the system of inequalities from Part a.

c. What number of talk-time minutes must Jovan stay below to ensure a charged cell phone for the entire trip?

30 Suppose the manufacturing company that supplies a chain of Packaging Plus stores receives a rush order for 290 boxes. It needs to fill the order in 8 hours or less.

- The factory has a machine that can produce 30 boxes per hour and costs \$15 per hour to operate.
- The factory can also use two student workers from less urgent tasks. Together, those students can make 25 boxes per hour at a cost of \$10 per hour.

What combination of machine and student work times will meet the deadline with least total cost?

31 The Dutch Flower Bulb Company bags a variety of mixtures of bulbs. There are two customer favorites. The Moonbeam mixture contains 30 daffodils and 50 jonquils, and the Sunshine mixture contains 60 daffodils and 20 jonquils.

The company imports 300,000 daffodils and 260,000 jonquils for sale each year. The profit for each bag of Moonbeam mixture is \$2.30. The profit for each bag of Sunshine mixture is \$2.50. The problem is deciding how many bags of each mixture the company should make in order to maximize profit without exceeding available supplies.

Explore how to use the inequality graphing capability of a CAS like that in *CPMP-Tools* to help find the combination of Moonbeam and Sunshine bags that give maximum profit.

32 When architects design buildings, they have to consider many factors. Construction and operating costs, strength, ease of use, and style of design are only a few.

For example, when architects designing a large city office building began their design work, they had to deal with the following conditions.

- The front of the building had to use windows of traditional style to complement the surrounding historic buildings. There had to be at least 80 traditional windows on the front of the building. Those windows each had an area of 20 square feet and glass that was 0.25 inches thick.
- The back of the building was to use modern-style windows that had an area of 35 square feet and glass that was 0.5 inches thick. There had to be at least 60 of those windows.
- In order to provide as much natural lighting for the building as possible, the design had to use at least 150 windows.

a. One way to rate the possible designs is by how well they insulate the building from loss of heat in the winter. The heat loss R in BTU's (British Thermal Units) per hour through a glass window can be estimated by $R = \frac{5.8A}{t}$, where A stands for the area of the window in square feet and t stands for the thickness of the glass in inches.

   i. What are the heat flow rates of the traditional windows and the modern windows?

   ii. Without graphing the number of window constraints, find the combination of traditional and modern windows that will minimize heat flow from the building.

b. Minimizing construction cost is another consideration. The traditional windows cost \$200 apiece, and the modern windows cost \$250 each. Without graphing the number of window constraints, find the combination of traditional and modern windows that will minimize the total cost of the windows.

c. Now consider the constraints graphically.

i. Write the constraint inequalities that match this situation and identify the feasible region.

ii. Write the objective functions for minimizing heat flow and for minimizing construction costs. Find the combination of traditional and modern windows that will minimize heat flow and total cost of the windows.

iii. Compare your responses to the minimization questions in Parts a and b to your region and objective functions. Adjust your solutions if necessary.

33 Graph the feasible regions for the following sets of inequalities in the first quadrant. A graphing calculator or CAS may be helpful in finding the "corner points" of the region.

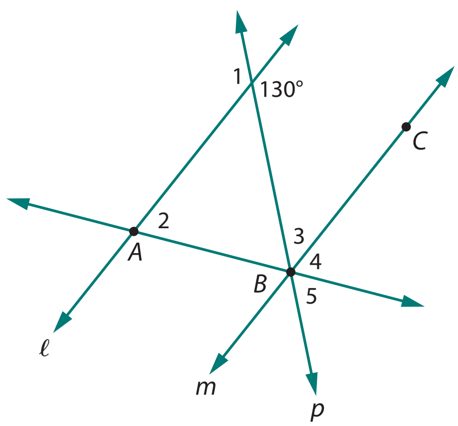
a. $\begin{cases} y \leq 10 - 3x \\ y \leq x^2 - 4x + 5 \end{cases}$

b. $\begin{cases} y \leq \frac{1}{x^2} \\ x + y \leq 4 \end{cases}$

c. $\begin{cases} y \geq 10(0.5^x) \\ y \geq 8 - 2x \end{cases}$

Review

34 In the diagram below, $\ell \parallel m$ and p bisects $\angle ABC$. Find the measure of each numbered angle.



35 Rewrite each expression in equivalent factored form.

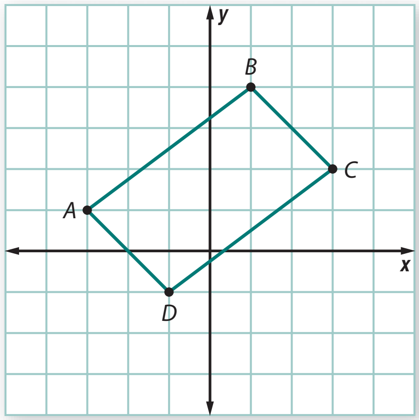
a. $x^2 - 16$

b. $x^2 + 10x + 25$

c. $x^2 + 6x - 40$

d. $2x^2 + 7x + 3$

36 Consider quadrilateral *ABCD* shown in the diagram below. The scale on both axes is 1.



a. Explain why quadrilateral *ABCD* is *not* a rectangle.

b. Find the area of quadrilateral *ABCD*.

c. Quadrilateral *A′B′C′D′* is the image of quadrilateral *ABCD* under a size transformation of magnitude 2 centered at the origin. What is the coordinate matrix of quadrilateral *A′B′C′D′*?

d. How are the angle measures of quadrilateral *A′B′C′D′* related to the angle measures of quadrilateral *ABCD*?

e. Find the area of quadrilateral *A′B′C′D′*.

37 Cesar entered a summer reading challenge at his local library. A winner would be determined as the child who read the most pages during the summer. Each week, Cesar reported the minutes he had spent reading. The librarian told him his ranking and his percentile. After the fourth week, Cesar was ranked 52nd and was at the 94th percentile. How many people were entered in the summer reading contest?

38 Determine if each pair of algebraic expressions are equivalent. If they are equivalent, provide algebraic reasoning to prove your claim. If they are not equivalent, provide a counterexample.

a. $(x + b)^2$ and $x^2 + b^2$

b. $8x - 2(x - 4)^2$ and $32 - 2x^2$

c. $\frac{6x + 4}{4}$ and $1.5x + 1$

39 Rewrite each expression with the smallest possible integer under the radical sign.

a. $\sqrt{84}$

b. $\sqrt{160}$

c. $25\sqrt{18}$

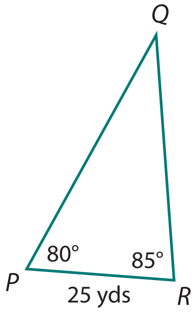
d. $(3\sqrt{5})^2$

e. $\sqrt{\frac{125}{9}}$

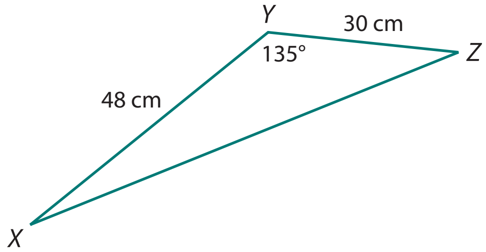
f. $\sqrt{20a^2}$, $a \geq 0$

40 For each triangle below, find the measures of the remaining side(s) and angle(s).

a.



b.



41 Consider the following two sets of data.

- the daily low temperature in International Falls, Minnesota, each day during the month of February
- the temperature of your math classroom at noon each day during the same month

a. Which set of data would you expect to have the greater mean? Explain your reasoning.

b. Which set of data would you expect to have the greater standard deviation? Explain your reasoning.

LESSON 3

Looking Back

The lessons in this unit involved situations in which important relationships among variables could be expressed by inequalities in one and two variables. In Lesson 1, you combined numeric, graphic, and algebraic methods for representing quadratic functions and solving quadratic equations in order to solve inequalities in one variable. You learned how to use inequality symbols, number line graphs, and interval notation to describe the solutions of those inequalities. In Lesson 2, you used graphs to identify the solutions for linear inequalities and systems of inequalities in two variables. You applied that knowledge to develop graphic and algebraic strategies for solving linear programming problems, optimizing a linear objective function given several linear constraints on the variables.

The tasks in this final lesson will help you review and organize your knowledge about solving inequalities and linear programming problems.

1 For each inequality below:

- make a sketch to show how the functions and constants in the inequality are related.
- use algebraic reasoning to locate the key intercepts and points of intersection.
- combine what you learn from your sketch and algebraic reasoning to solve the inequality.
- describe each solution set using symbols, a number line graph, and interval notation.

a. $8 - x^2 \leq 6$ **b.** $x^2 + 4x + 4 < -3x - 8$

c. $7 + x > 2 + 3x - x^2$ **d.** $3 - x^2 \geq x^2 + 5$

2 Write inequalities that express the same information as the interval notation in these situations.

a. $[3, 6]$ **b.** $(-1, 4)$

c. $(-7, 1.5]$ **d.** $(-\infty, -2) \cup [7, \infty)$

3 For each condition described below, write an inequality involving one linear function and one quadratic function that satisfies the condition. Then draw a graph that illustrates the relationship between the two functions.

a. The inequality has no solutions.

b. All real numbers are solutions of the inequality.

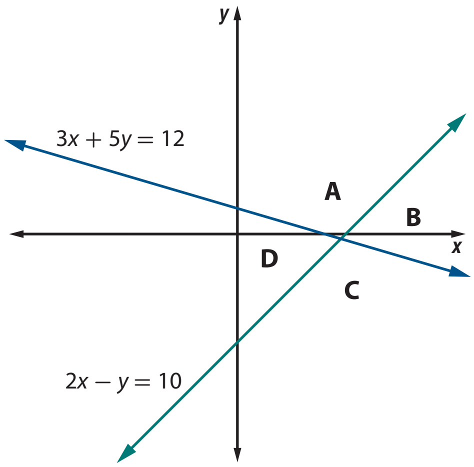
c. Exactly one value satisfies the inequality.

4 Suppose that Lee wants to buy macadamia nuts and banana chips for an after-school snack. Both are available in the bulk foods section of a local grocery store. Macadamia nuts cost \$8 per pound, and banana chips cost \$5 per pound.

a. If Lee has \$4 to spend, write an inequality whose solution set gives the (*pounds of macadamia nuts m*, *pounds of banana chips b*) combinations that Lee can buy.

b. Draw a graph that uses shading to show the region of the coordinate plane that contains all points (m, b) that satisfy the inequality.

5 The graphs representing the system of linear equations $\begin{cases} 3x + 5y = 12 \\ 2x - y = 10 \end{cases}$ are shown to the right. How could you replace each of the "=" symbols in the two equations with a "<" or ">" symbol so that the solution of the resulting system of inequalities would be:



- region A
- region B
- region C
- region D

6 A small sporting goods manufacturer produces skateboards and in-line skates. Its dealers demand at least 30 skateboards per day and 20 pairs of in-line skates per day. The factory can make at most 60 skateboards and 40 pairs of in-line skates per day.

a. Write inequalities expressing the given constraints on daily skateboard and in-line skate manufacturing.

b. Graph the feasible region. Label each segment of the boundary of the region with the equation of that line.

c. How many combinations of numbers of in-line skates and skateboards are possible?

d. Suppose the total number of skateboards and pairs of in-line skates cannot exceed 90.

i. What inequality expresses this constraint?

ii. Show this new constraint on your graph.

e. Find coordinates of the corners for the new feasible region.

f. Suppose the profit on each skateboard is \$12 and on each pair of in-line skates is \$18. Write the profit function.

g. How many of each product should the company manufacture to get maximum profit?

Summarize the Mathematics

In this unit, you investigated problems that could be solved by writing and reasoning with inequalities in one or in two variables. In some cases, the problem could be represented with a single inequality. In other cases, as in linear programming problems, representation of the problem required a system of constraint inequalities and an objective function.

a Consider the two functions $f(x)$ and $g(x)$. Describe a general strategy for finding values of x for which each of the following is true.

i. $f(x) = g(x)$ **ii.** $f(x) < g(x)$ **iii.** $f(x) > g(x)$

b The inequality $x^2 - 5x - 24 > 0$ is true for values of x that are less than -3 or greater than 8. Describe these values of x using:

i. inequality symbols **ii.** a graph on a number line **iii.** interval notation

c In what ways is solving an inequality like $x^2 - 5x - 24 > 0$ different from finding the solution set for one like $y > x^2 - 5x - 24$? How is it the same?

d What steps are involved in graphing the solutions of an inequality like $3x + 5y \leq 10$?

e Describe the roles played by the following elements of linear programming problems.

i. constraints **ii.** feasible region **iii.** objective function

f In a linear programming problem, where in the feasible region will the objective function have its optimum (maximum or minimum) value? Why is that true?

Be prepared to explain your ideas and methods to the class.

Check Your Understanding

Write, in outline form, a summary of the important mathematical concepts and methods developed in this unit. Organize your summary so that it can be used as a quick reference in future units and courses.